

Mathematics 4301 – Abstract Algebra

Student Learning Outcomes

- 1. The students will demonstrate factual knowledge including the mathematical notation and terminology used in this course.** Students will read, interpret, and use the vocabulary, symbolism, and basic definitions used in abstract algebra, including binary operations, relations, groups, subgroups, homomorphisms, rings, and ideals.
- 2. The students will describe the fundamental principles including the laws and theorems arising from the concepts covered in this course.** Students will develop and apply the fundamental properties of abstract algebraic structures, their substructures, their quotient structure, and their mappings. Students will also prove basic theorems such as Lagrange's theorem, Cayley's theorem, and the fundamental theorems for groups and rings.
- 3. The students will apply course material along with techniques and procedures covered in this course to solve problems.** Students will use the facts, formulas, and techniques learned in this course to prove theorems about the structure, size, and nature of groups, subgroups, quotient groups, rings, subrings, ideals, quotient rings, and the associated mappings. Students will also solve problems about the size and composition of subgroups and quotient groups; the orders of elements; isomorphic groups and rings; and the composition of ideals.
- 4. The students will apply course material along with techniques and procedures covered in this course to solve problems.** Students will use the facts, formulas, and techniques learned in this course to prove theorems about the structure, size, and nature of groups, subgroups, quotient groups, rings, subrings, ideals, quotient rings, and the associated mappings. Students will also solve problems about the size and composition of subgroups and quotient groups; the orders of elements; isomorphic groups and rings; and the composition of ideals.

Course Content

Textbook: *Introduction to Abstract Algebra, Theory* by D.S. Malik, John N. Mordeson, M.K. Sen. Available with permission from

<https://people.creighton.edu/~dsm33733/MTH581/Introduction%20to%20Abstract%20Algebra.pdf>

- 1. Binary Operations:** Definition and examples, properties.
- 2. Groups:** Definition, examples, finite and infinite groups, abelian groups, properties.
- 3. Subgroups:** Definition, generators and defining relations.
- 4. Functions:** Injections, surjections, bijections, compositions, inverses.
- 5. Permutations:** Symmetric groups, dihedral groups, cycles and transpositions, even and odd permutations, alternating groups.
- 6. Isomorphisms and Homomorphisms:** Definitions and properties, normal subgroups, kernel and range, Cayley's Theorem.
- 7. Cyclic Groups:** Finite and infinite cyclic groups, subgroups.
- 8. Partitions and Equivalence Relations**
- 9. Quotient Groups:** Lagrange's Theorem, examples and applications.
- 10. Rings, Fields, and Integral Domains:** Commutative rings, rings with unity, invertible elements, zero divisors.

Additional topics to be covered as time permits: The Fundamental Homomorphism Theorem, the Correspondence Theorem, Cauchy's Theorem.