

## Mathematics 4301 – Abstract Algebra

### Student Learning Outcomes

- 1. The students will demonstrate factual knowledge including the mathematical notation and terminology used in this course.** Students will read, interpret, and use the vocabulary, symbolism, and basic definitions used in abstract algebra, including binary operations, relations, groups, subgroups, homomorphisms, rings, and ideals.
- 2. The students will describe the fundamental principles including the laws and theorems arising from the concepts covered in this course.** Students will develop and apply the fundamental properties of abstract algebraic structures, their substructures, their quotient structure, and their mappings. Students will also prove basic theorems such as Lagrange's theorem, Cayley's theorem, and the fundamental theorems for groups and rings.
- 3. The students will apply course material along with techniques and procedures covered in this course to solve problems.** Students will use the facts, formulas, and techniques learned in this course to prove theorems about the structure, size, and nature of groups, subgroups, quotient groups, rings, subrings, ideals, quotient rings, and the associated mappings. Students will also solve problems about the size and composition of subgroups and quotient groups; the orders of elements; isomorphic groups and rings; and the composition of ideals.
- 4. The students will develop specific skills, competencies and thought processes sufficient to support further study or work in this or related fields.** Students will acquire a level of proficiency in the fundamental concepts and applications necessary for further study, including graduate work, in academic areas requiring abstract algebra as a prerequisite, or for work in occupational fields requiring a background in abstract algebra or other highly abstract mathematics. These fields might include the physical sciences and engineering as well as mathematics.

### Course Content

**Textbook:** *Elements of Abstract and Linear Algebra*, by E. H. Connell. The textbook is available at <http://www.math.miami.edu/~ec/book/>.

- 1. Fundamentals.** Sets, mappings, properties of composite mappings, binary operations, relations.
- 2. The Integers.** Mathematical induction, congruence of integers, congruence classes.
- 3. Groups.** Definition of a group, subgroups, cyclic groups, isomorphisms, homomorphisms.
- 4. More on Groups.** Finite permutation groups, Cayley's theorem, normal subgroups, quotient groups.
- 5. Rings, Integral Domains, and Fields.** Definition of a ring, integral domains and fields, the field of quotients of an integral domain.
- 6. More on Rings.** Ideals and quotient rings, ring homomorphisms, the characteristic of a ring.

**Additional Content (as time allows).** Direct sums, finite abelian groups, maximal ideals, polynomials over a ring.