Mathematics 4311 -- Numerical Analysis

Student Learning Outcomes

1. **The student will demonstrate factual knowledge including the mathematical notation and terminology used in this course.** Students will read, interpret, and use the vocabulary, symbolism, basic definitions used in numerical analysis including those related to topics learned in calculus and algebra and revisited in this course; limits, continuity, numerical integration, numerical differentiation, ordinary differential equations, systems of linear equations, and polynomial interpolation.

2. **The students will describe the fundamental principles including the laws and theorems arising from the concepts covered in this course.** Students will identify and apply the properties and theorems that result directly from the definitions as well as statements discovered in calculus and extended in this course; for example, Rolle's Theorem, Mean Value Theorem, Intermediate Value Theorem, Taylor series, theorems on convergence and existence and their error terms.

3. **The students will apply course material along with techniques and procedures covered in this course to solve problems**. Students will use the facts, formulas, and techniques learned in this course to develop and use algorithms and theorems to find numerical solutions and bounds on their error to various types of problems including root finding, polynomial approximation, numerical differentiation, numerical integration.

4. **The students will develop specific skills, competencies, and thought processes sufficient to support further study or work in this field or related fields**. Students will gain the ability to use a software package such as Maple to solve numerical problems and acquire a level of proficiency in the fundamental concepts and applications necessary for further study in academic areas requiring numerical analysis as a prerequisite for graduate work or for work in occupational fields. These fields might include further study in mathematics, engineering, computer science, or the physical sciences.

Course Content

Textbook: Numerical Analysis, Eighth Edition, by R. Burden – J. Faires. The following chapters including the particular sections listed are covered.

1. **Mathematical Preliminaries and Error Analysis.** Review of Calculus (including limits, derivatives, continuity, Rolle's Theorem, Mean Value Theorem, Extreme Value Theorem, Intermediate Value Theorem, and extensions of these theorems, Taylor and McLaurin Series), Round-off Errors and Computer Arithmetic, Algorithms and Convergence, Numerical Software.

2. **Solutions of Equations in One Variable.** The Bisection method, Fixed-Point Iteration, Newton's Method, Error Analysis for Iterative Methods, Accelerating Convergence.

3. **Interpolation and Polynomial Approximation**. Interpolation and the Lagrange Polynomial.

4. **Numerical Differentiation and Integration.** Numerical Differentiation, Richardson's Extrapolation, Elements of Numerical Integration, Composite Numerical Integration.

6. **Direct Methods for Solving Linear Systems.** Linear Systems of Equations, Pivoting Strategies.

7. **Iterative Techniques in Matrix Algebra.** Norms of Vectors and Matrices, Eigenvalues and Eigenvectors, Iterative Techniques for Solving Linear Systems, Error Bounds and Iterative Refinement.

9. **Approximating Eigenvalues.** Linear Algebra and Eigenvalues, The Power Method, Householder's Method, The QR Algorithm.

Additional Content

(Additional topics as time permits and as to the interest of the instructor.)

3. **Interpolation and Polynomial Approximation**. Divided Differences, Hermite Interpolation, Cubic Spline Interpolation.

4. **Numerical Differentiation and Integration.** Romberg Integration, Adaptive Quadrature Methods, Gaussian Quadrature.

5. **Initial – Value Problems for Ordinary Differential Equations.** Elementary theory of Initial-Value Problems, Euler's Method, Higher-Order Taylor Methods, Runge-Kutta methods, Error Control and the Runge-Kutta-Fehlberg Method, Multistep Methods.

6. **Direct Methods for Solving Linear Systems.** Linear Algebra and Matrix Inversion, The Determinant of a Matrix, Matrix Factorization, Special Types of Matrices.

7. **Iterative Techniques in Matrix Algebra.** The Conjugate Gradient Method.

8. **Approximation Theory.** Discrete Least Squares Approximation, Orthogonal Polynomials and Least Squares Approximation, Chebyshev Polynomials and Economization of Power Series, Rational Function Approximation, Trigonometric Polynomial Approximation, Fast Fourier Transforms.

Also: A. Monte Carlo Methods and Simulation. Random numbers, estimation of areas and volumes, simulations. **B. Further Topics in Numerical Linear Algebra.** Review of Gram-Schmidt orthogonalization, QR Factorization, Singular Value Decomposition, applications.