Math 1303 Plane Trigonometry (Measurement of Triangles)

Given a triangle ABC – meaning that the vertices(and angles) are labeled as A, B, and C respectively – the sides are naturally labeled in terms of the angle opposite them; a, b, and c.

Is this a right triangle? ____________

Can $a = 30$ inches, $b = 24$ inches, and $c = 50$ inches? Why or why not?

**True or False**

$\pi = 3.14$  

1 is larger than 0.9999...  

$\sqrt{2} = 1.41$

What do you see – picture? ____________  

Advise:

1. **Learn(memorize) definitions and identities.** It is a necessary evil that will help you understand basic concepts.

2. **Ask for help immediately.** No question is too small, no question is ever dumb – not asking is.

3. **Homework** – the first step of success is an attempt to understand.

4. Use math lab, use my office hours

Help is available now – **not ten minutes** before a test (unless it is a minor idea), **not 2 hours before a final exam**

Your thoughts (definition) of what we are about to study --- Trigonometry:

Fill in the Blank:

1. I expect to make the following letter grade in class ______

2. I will accomplish this by studying _________ hours per week for this class.

3. I expect to attend math lab (MCS 215) (M-Th 2:00 – 5:00, F 2:00-4:00) _________ times before each exam

4. I plan to visit Mr. Montemayor in his office _________ times in his office during the semester for help or because he is a pleasant person to talk to.

5. I understand that grades of A, B, C are mostly determined by my test grades. A grade can be improved by HW grades. Failure to attempt HW will significantly decrease my grade.

__________ (initial)
Introduction and review of ideas
geometric properties; point, ray, angle, measurement of angles, triangles

Geometry:
point (P), \( \overrightarrow{AB} \), \( \overleftarrow{AB} \)

angle: vertex
initial side, terminal side

We write an angle as: \( \angle A, B \), \( \angle \), or ________

Def. Given a circle we say an angle is a central angle if its vertex is at the center of the circle.

measure of an angle:
look at two ways of measuring an angle

radian measure: to be discussed later for now \( \angle \) Given a circle of any radius \( r \), construct a central angle so the intercepted arc of the circle is also of measure \( r \) we say the central angle has a measure of 1 radian.

\[ \begin{array}{c}
\text{degree:} \\
\text{Given a circle of any radius – construct a central angle so that 360 of them (of equal length) make up the inside of the circle} \\
\text{Each angle is said to be of measure 1 degree, } 1^\circ
\end{array} \]

From Geometry: angles do not really have a sign – the opening (the angle) has a certain measure

A trigonometric angle has a sign:
It is considered to be positive if measured counter clockwise, negative if measured clockwise
Trigonometric angles:

1) draw angle ACB so that its measure is $+225^\circ$

2) Draw an angle F so that its measure is $-40^\circ$

Types of angles (with respect to its measure):

- right: ______
- straight: ______
- acute: _________
- obtuse: _______

A pair of acute angles are said to be **complementary** if $\theta$ and $90^\circ - \theta$ are ________

Positive angles are said to be **supplementary** if $180^\circ - \beta$ and $\beta$ are ________

For example:

- $30^\circ$ and $60^\circ$ are _________
- $110^\circ$ and $70^\circ$ are _________
- $-20^\circ$ and $110^\circ$ are _________

**Note:** we can also say that

$\theta$ and $90^\circ - \theta$ are ________

$180^\circ - \beta$ and $\beta$ are ________

**If** angle A and B are complementary *with* A and C supplementary, *what can you say about B and C?*

$C + B = ________$

$C - B = ________$

**Properties of triangles:**

- Sum of the lengths of any two sides must exceed the length of the third

- Can you construct a triangle that has lengths: 30, 10, and 12 inches

- Angles of a triangle must add up to ______ degrees
When we look at polygons we can determine the sum of the measure of the

a) interior angles:

triangles: _________ 
quadrilaterals: _________ 
in general n-sided polygon: _______

b) exterior angles:

Similar:
figures are considered similar if corresponding angles have the same measure (congruent)
copies that have been enlarged, contracted,...

triangles are similar if:

Congruent
figures are considered congruent if corresponding angle have the same measure and corresponding sides are of equal length

exact copies that have been rotated, shifted,...

triangles are congruent if:

Are congruent triangles also similar triangles? ____________

Are similar triangles congruent? ____________
Properties of similar triangles (and in turn – congruent):
the following examples are right triangles but the conclusions are not limited to right triangles.

1) ratio of corresponding sides are equal:
   Notice the ratio of corresponding sides of triangles ABC and DEF -

2) ratio of corresponding ht’s are also equal (to ratio of corresponding sides)
   What is the ht. of the first triangle? _____

3) What about perimeters and areas? Any relationship? Use the example at the top of the page to come up with a conclusion?
   a) Perimeter           b) area

Other Examples:

Special Triangles:
right triangle: triangle with one angle that of measure ____________
   acute triangle: triangle in which all angles have a measure __
   obtuse triangle: triangle with one angle of measure ____________
From Algebra:

Given any two points on a coordinate system – we can find the distance between them.

If the points are represented by \( A(x_1, y_1) \) and \( B(x_2, y_2) \), then we can find the distance \( d \) between them by

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Example: (Help in understanding the formula above)

A boat is sitting at sea level facing a cliff that is 300 feet above sea level. If the cliff is 500 feet high, then what is distance from the boat to the top of the cliff (airline distance) ?

Definition:

Let \( c \) be a real number. We define the absolute value of \( c \), written \( |c| \), by

\[
|c| = \begin{cases} 
  c & \text{if } c \geq 0 \\
  -(c) & \text{if } c < 0
\end{cases}
\]

examples:

\[
| - \frac{2}{3} | = \quad - | - 2 | - \quad | -3 - (-5) | =
\]

\[
\frac{x}{|x|} = \quad | 5 - \sqrt{17} | =
\]
Pythagorean Theorem:
Given a right triangle $ABC$ with corresponding sides $abc$, we find that
\[ a^2 + b^2 = c^2 \]

A better way to state this might be:

**Given triangle $ABC$, we say that the triangle is a right triangle if and only if** $(\text{iff})$ \[ a^2 + b^2 = c^2 \]

**examples:**

1) Construct the distance formula between two points.

2) A 15 foot tree will need three wire supports to keep it from falling down. The supports will be placed 9 feet above the ground at a distance of 12 feet from the ground. How much wire will be need to create the supports?

   answer: at least ______

3) **A Construction: Proof of Pythagorean Thm. (Proof on page 6 of your text)**

   ![Diagram of a square and triangles illustrating the proof of the Pythagorean Theorem]

   ex. Given any right triangle $ABC$ (We will label $C$ as the right angle). Find the missing sides

   ![Diagram of two right triangles with sides labeled 12, 15, and $x$ on the left, and 5, 4, and $x$ on the right]
Note:
Does the following triangle represent a right triangle or not?

![Triangle Diagram]

Explain your reasoning:

Ex. A ski lift is 4000 feet long (assume it forms a straight line). If the vertical rise of the lift is half the horizontal length, then how high does it move from beginning to end?

To begin learning about the definitions of triangles and their properties we study some special angles (triangles).

1) axes angles:

\[ 90^\circ, 180^\circ, 270^\circ, 360^\circ, 0^\circ, -270^\circ, \ldots \]

Find points on these angles when initial side is on positive x-axis and vertex is at origin.

A point on

\[ 180^\circ \rightarrow \underline{______} \quad 360^\circ \rightarrow \underline{______} \quad (-270^\circ) \rightarrow \underline{______} \]

2) \(30^\circ - 60^\circ - 90^\circ\) right triangle and

Begin with an equilateral triangle:

from geometry this tells that all angles are of equal length

Each side being of equal length tells once we choose the length of one side → choose 2x – all other sides are known.

Use another property from geometry:
We are more comfortable with nice “simple numbers”. Maybe instead of $2x$ – we can use 2 and make a similar construction.

3) the $45^\circ - 45^\circ - 90^\circ$ right triangle
   With a little bit geometry – we can come up with the following result.
   From Geometry:
   If two angles are equal (congruent),
   then their opposite sides are also of equal length.
   If we bisect the third angle, then the opposite side is also bisected.

Again, instead of using $2x$ we can use the 2 as the length of the congruent sides.

Examples:
1. How high above the ground can a ladder reach if it is 20 feet long and it leans against a wall making a $30^\circ$ with the ground. For safety purposes. ?

2. Another Example:
   A large cylinder leans against a wall. The cylinder is of radius 4 feet.
   What is the length of the pipe that will go over the cylinder (tangent) and touch the wall 12 feet above the ground.

   Is there enough information?
Solving a triangle means to find all of the sides and all of the angles. Assume the following two triangles are right triangles.

Examples:

Find the missing side.

\[
\begin{align*}
\text{a} & \quad 2a \\
\text{x} & \\
\end{align*}
\]

\[x = \ldots\]

\[
\begin{align*}
\text{2t} & \\
\text{x} & \\
\end{align*}
\]

\[x = \ldots\]

Find the length of the diagonal of the following “square pond” that is 40 feet wide and 40 feet long

A person stands at the edge of a river. He looks across to a point on the other side of the river. Find the width of the river from the following picture.

River

\[
\begin{align*}
\text{WX} & = 40 \text{ feet, angle } X = 60^\circ \\
\end{align*}
\]

Find the missing side of each right triangle.

\[
\begin{align*}
12 & \quad r \\
\text{r} & \\
\end{align*}
\]

\[r = \ldots\]

\[
\begin{align*}
24 & \quad 60^\circ \\
y & \\
\end{align*}
\]

\[y = \ldots\]
Tell me what property each of the following triangles have in common

(c.s)

(s.t.)

More Algebra Review:

Sect. 1.2 The rectangular coordinate system.

Directed line, directed line segment

Point P(x,y): x coordinate or the abscissa, y coordinate or the ordinate.
Radius vector:
the distance from origin to Point P, we choose this distance to have positive measure

Given a point P(2, 7) find the radius vector of P (from the origin to point P)
Notice the signs on the x and y coordinates in each quadrant.

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<tr>
<th>QI:</th>
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<th>QIII:</th>
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<td>x</td>
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<td>r</td>
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<tr>
<td>y/r</td>
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<tr>
<td>x/y</td>
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</tbody>
</table>

To help us sketch curves we may use x-intercept, y-intercept.

x-intercept: let y = 0 solve for x – this is called the x-intercept
y-intercept: let x = 0 solve for y – this is called the y-intercept

ex. 2x – y = 4

ex. Sketch the graph of y = x^2 + x

More Algebra Review:

Factoring:

1) GCF: x^2 – 4x =

2) trinomial: x^2 + 4x – 5 =

3) Difference of Squares: 4x^2 - 9 =

4) Sum/Difference of cubes: x^3 + 8y^3 =

1 – 27y^3 =

ex. Factor: sin^2 x - cos^2 x =

tan x + tan x • sec x =

e^x + e^{2x} =

|x|^2 – 4 =
Solving Equations, fractions: Factor, find common denominators

1) find the solution of \( \frac{6x^2 - 1}{2x - 2} = 3x \)

2) combine by getting a common denominator \( \frac{2x}{\sin x} + \frac{3}{\cos x} = \text{________} \)

3) Perfect Squares: \((x + 2y)^2 = \text{________} \)

\[ \left( \frac{2^x - 2^{-x}}{2} \right)^2 = \text{________} \]

Quadratic Equations, Quadratic Formula:

\[ x^2 - 4x = \text{________} \quad \quad \quad 9x^2 - 4 = 0 \rightarrow x = \text{________} \]

Quadratic Formula:
**Brief Introduction to Relations and Functions**

Def. A relation is a correspondence between two sets.

The first set makes up the domain of the relation – permissible values of the relation. The second set is called the range of the relation – the results (output) of using the permissible values.

ex.

We can write them in relation form:

\[ R = \{ ______________________ \} \quad R = \______________________ \]

Both of these are examples of relations. The first one is also an example of a function.

Because relations involve two sets – we can think of them in terms of x’s and y’s.

Domain: consists of values of x  
Range: consists of values of y

**Def.** A function is a special kind of relation in which each value of x corresponds to a single value of y. (one x produces one value of y)

We can define a function and a relation in terms of ordered pairs.

Def. A function is a relation in which no two ordered pairs of points have the same 1st coordinate but different 2nd coordinates.
If we are given graphs, we could use this idea to **introduce the vertical line test**

**From Algebra:**

\[
\begin{align*}
f(x) &= 2x - 3 \\
g(x) &= x^2 + x \\
x^2 + y^2 &= 16
\end{align*}
\]

have graphs:

Note: vertical line test

All functions are relations but not all relations are functions.

Now that we have been re-introduced to the idea of functions and relations →

**Trigonometry.**

As we said before - there is a difference between a geometric angle and a trigonometric angle.

The opening between these two rays: 20°.

We can talk about the angles; 380°, -340°, 20° and how they relate to each other.

**Def. (standard position).**

An angle θ is said to be in standard position provided its vertex is at its origin and its initial side is on the positive (side) x-axis.

examples:
Note: An angle in standard position is said to be in quadrant I if its terminal side is in quadrant I. Same for other quadrant angles.

examples:

True or False.
Every quadrant I angle is acute. ___________
Every quadrant I angle is positive. ___________
Every acute angle is a quadrant I angle. __________

Note: If an angle is in standard position and its terminal side lies on one of the axes, we call it an axis angle.

ex. 0°, 90°, 180°, ..., -90°, -180°, ...

Def (coterminal) Let θ and β be any two given angles in standard position.
We say that θ and β are coterminal if they have the same terminal side.

examples:

Find two angles that are coterminal with 100°. How many angles are coterminal with 100°?

Note: 200° and -160° are coterminal but not equal neither are 10° and 370°.
( may have a similar algebraic result with respect to the functions we are going to study)
Now we are ready to define the six trigonometric functions – ratios of sides of a right triangle.

**Def.** Let \( P(x,y) \) be a point on a plane. Let \( r \) represent the radius vector of \( P \) and \( \theta \) be the standard position angle with terminal side passing through \( P \). We define the six trigonometric functions as follows;

\[
\begin{align*}
\sin \theta &= \frac{y}{r}, & \csc \theta &= \frac{r}{y}, \\
\cos \theta &= \frac{x}{r}, & \sec \theta &= \frac{r}{x}, \\
\tan \theta &= \frac{y}{x}, & \cot \theta &= \frac{x}{y}
\end{align*}
\]

Those of you that are familiar with the definitions you may remember that there is another way to define them – that’s coming. -- but those definitions involve triangles. The definitions above can be used with just angles.

**Note:**
- \( \theta \) (the angle) is part of the definition
- \( \sin, \cos, \tan \): have no meaning by themselves
- We get lazy and use them as such but we should include the angle

**All we need to find the six trig. functions are \( P(x,y) \) and \( r \).**

**ex.** Let \( P(12, 5) \) be a point on the terminal side of \( \theta \), where \( \theta \) is a standard position angle. Find the value of each of the six trigonometric functions.

\[
\begin{align*}
\sin \theta &= \\
\cos \theta &= \\
\tan \theta &=
\end{align*}
\]

\[
\begin{align*}
csc \theta &= \\
sec \theta &= \\
cot \theta &=
\end{align*}
\]

**What if we had chosen a different point, say \( Q(24, 10) \) ?**

First, is \( Q \) on the terminal side of \( \theta \) – or is this an entirely different angle?
Note: Does it matter what point we use on the terminal side of $\theta$?

ex. Let $P(3, 4)$ and $Q(12, 16)$ be points on the terminal side of some angle $\beta$. Look at the trig functions of $\beta$.

ex. Find the six trig. functions of axis angles:

<table>
<thead>
<tr>
<th>Angle</th>
<th>$\sin \theta$</th>
<th>$\cos \theta$</th>
<th>$\tan \theta$</th>
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<tbody>
<tr>
<td>$180^0$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$720^0$</td>
<td>0</td>
<td>1</td>
<td>0</td>
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</tbody>
</table>

ex. Other special angles

<table>
<thead>
<tr>
<th>Angle</th>
<th>$\sin \theta$</th>
<th>$\cos \theta$</th>
<th>$\tan \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30^0$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{3}}{3}$</td>
</tr>
<tr>
<td>$45^0$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>1</td>
</tr>
<tr>
<td>$60^0$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\sqrt{3}$</td>
</tr>
</tbody>
</table>

Your author has some nice ways to remember these functions – feel free to look at them on page ?

Or we can use the following right triangles to answer the
Notice that r is always positive, x is positive in quadrant I and IV, and y is always positive in quadrant I and II. So, what sign does each trig. function have in each quadrant

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<tr>
<td>sin $\theta$</td>
<td>$y/r$</td>
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<tr>
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<tr>
<td>tan $\theta$</td>
<td>$y/x$</td>
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ex. Given $\cos \theta = -3/5$ and $\theta$ is in quadrant III find the remaining trig functions of $\theta$.

$\sec \theta = \_\_\_\_$  $\sin \theta = \_\_\_\_$  $\csc \theta = \_\_\_\_$  

$\tan \theta = \_\_\_\_$  $\cot \theta = \_\_\_\_$
Consequences of the definitions:

1. (reciprocal functions) - because of the way the trigonometric functions are defined

   we can pair them as reciprocal functions:

   __________________ and ___________________ →

   __________________ and ___________ →

   ____________________ and __________ →

2. Any trig. function of an angle is equal to the same trig. function of all angles coterminal with it.

   (as long as the trig. function is defined at that angle)

   ex. $\sin 20^\circ = \sin 380^\circ = \sin (-340^\circ)$ $\sec \theta = \sec (360^\circ + \theta)$ $\tan \beta = \tan (\beta - 360^\circ)$

3. The signs of the trig functions are now determined in each quadrant.

   S(students)     A (all )

      T(take )       C(calculus)

Also,

Def. We pair functions as follows and call each pair of functions cofunction of each other.

   $\sin \theta$ and $\____________$ $\tan \theta$ and $\____________$ $\sec \theta$ and $\____________$

   Notice: They are not equal if the same angle is used. They are called cofunctions.

4. Any trigonometric function of an acute angle is equal to the cofunction of its complementary angle.

   ex. $\sin 30^\circ = \cos $ $\sec 1^\circ = \csc$

   if $\alpha$ is an acute angle, then $\tan \alpha = \cot$
Alternate definitions of trig. functions.

Given the following right triangle - find $\sin \alpha$. Answer ? _______

![Right Triangle Diagram]

What about $\tan \beta$? ________

What is the problem with these questions?

Draw an angle $\theta$ in standard form and label it with respect to $x,y,$ and $r$. Choose $\theta$ to be in quadrant I. We will allow $\theta$ to be in other quadrants later.

Also, label the sides as opposite, adjacent, and hypotenuse – with respect to $\theta$

![Graph with labeled sides]

Def. Define the trig functions in terms of the words opposite, adjacent, and hypotenuse with respect to some angle $\theta$.

$\sin \theta =$ _______ $\cos \theta =$ ____________ $\tan \theta =$ _______

$csc \theta =$ _______ $\sec \theta =$ ____________ $\cot \theta =$ _______

Example:

You are running on a flat dirt road and encounter a sharp decline on the road before it flattens out again. If the length of the decline on the road is 20 feet, then how long must you jump horizontally so that you clear the 20 foot path of road? Assume that the drop can be measured in degrees as well and it turns out that the road drops $30^\circ$.  


Ex. Without the use of a calculator – find each of the following.

1) \( \sin 70^\circ \cdot \csc 70^\circ - \cos^2 30^\circ = \) __________

2) \( 1 - 2 \sin^2 60^\circ = \) ______________

3) \( 1 + \tan 45^\circ = \) __________

4) \( \sqrt{3} \csc 60^\circ = \) ______________

5) \( \sin 60^\circ \cdot \csc 60^\circ + \tan 45^\circ - \cos 30^\circ = \) ______________

6) \( \sin^2 45^\circ + \cos^2 45^\circ - 1 = \) ________

7) Use the given triangles to find all six trig. functions.

\[ \begin{array}{c}
\text{A} \quad \text{B} \\
\text{15} \\
\text{112} \\
\text{40} \\
\text{41} \\
\text{9} \\
\end{array} \]

\[ \begin{array}{c}
\text{F} \quad \text{15} \\
\text{112} \\
\text{113} \\
\text{D} \\
\end{array} \]

sin \( A = \) _______  
sec \( B = \) _______  
sin \( F = \) _______

tan \( B = \) _______  
cot \( E = \) _______  
\( \cos E = \) _______

More Examples:

1) Which is larger - if \( \alpha \) and \( \beta \) are complementary with \( \alpha > \beta \) and \( \alpha \neq 0^\circ \) or \( 90^\circ \), \( \beta \neq 0^\circ \) or \( 90^\circ \)

\( \sin \alpha \) or \( \sin \beta \)

\( \sin \alpha \) or \( \cos \beta \)

\( \sin \alpha \) or \( \cos \alpha \)
Identities – conditional equations – contradictions

Identities:

\[ x^4 = x \times x \times x, \quad 4x = x + x + x + x \]

Find where the following lines intersect: \(2x - y = 3\) and \(4x - 2y = 6\) → 

Conditional Equations:

\[ 4 + x = 2 \rightarrow \]
\[ x^2 - 3 = 0 \rightarrow \]

Contradiction:

\[ x + 4 = x - 4, \quad \sin \theta \cdot \cos \theta = \tan \theta \rightarrow \]

Examples:

\[ \sin x = \frac{1}{2} \quad \frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1 \quad \frac{\tan^3 x + 1}{\tan x + 1} = \sec^2 x + 1 \]

Fundamental Trig. Identities

1. We have already mentioned \textbf{reciprocal functions}.

\[ \begin{array}{ccc}
1) & \csc \theta = & \_ \_ \_ \_ \_ \\
2) & \sec \theta = & \_ \_ \_ \_ \_ \\
3) & \cot \theta = & \_ \_ \_ \_ \_ \\
\end{array} \]

2. Also have \textbf{ratio identities} (4, 5), and

\[ \begin{array}{ccc}
4) & \tan \theta = & \_ \_ \_ \_ \_ \\
5) & \cot \theta = & \_ \_ \_ \_ \_ \\
\end{array} \]

3. \textbf{Pythagorean identities} (6, 7, 8).

\[ \begin{array}{ccc}
6) & & \\
7) & & \\
8) & & \\
\end{array} \]

These eight identities form what we call the fundamental trig. identities.
examples:

Find the reciprocal of -4/3 → ______________

If \( \sin \theta = 3/7 \), then what is \( \csc \theta \) ? ______________

What is \( \cos \theta \) ? ________ Why? ______________

If \( \sin \theta = -4/5 \) and \( \cos \theta = 3/5 \),

then what is \( \tan \theta \) ? ______________ What quadrant is \( \theta \) in? __________

Given \( \cos \theta = -3/7 \), What is \( \cos^2 \theta \) ? __________

What is \( \sin^2 \theta \)? __________

What is \( \sin \theta \)? __________

Given \( \cos A = -3/5 \), find

\[
\sin^2 A \cdot \frac{\sqrt{1 - \cos A}}{2} = __________
\]

Algebraic techniques with Trigonometric functions

a) \( \sin \theta \cdot \cot \theta = __________ \)  b) \( \sin^2 \theta / (1 - \cos \theta) = __________ \\

c) \( 1/\sin \theta + 1/\cos \theta = __________ \)  d) \( (2 \sin \theta - 1)^2 = __________ \\

To Prove Identities:

1) Work each side independently from each other – it is not an equation

2) use algebraic rules, other identities to convert from one form to another – common denominator , reduce fractions, factor,…

3) If need to , change in terms of sines and cosines .

NOTE: it is common not to change function in terms of x, y, and r to prove identities.

ex. Prove or Disprove

1) \( \cos \theta \csc \theta \tan \theta = 1 \)

2) \( \frac{\csc \theta \tan \theta}{\sec \theta} = 1 \)

3) \( \sec \theta - \cos \theta = \frac{\sin^2 \theta}{\cos \theta} \)

4) \( \frac{\sin^2 \theta - \cos^2 \theta - 2 \sin \theta}{3 \sin \theta + 1} = \frac{\sin \theta - 1}{\sin \theta - 1} \)
Def. We pair functions as cofunctions:

\[
sin \theta, \cos \theta; \quad sec \theta, \csc \theta; \quad tan \theta, \cot \theta
\]

Ex.

Find \( sin \alpha \). \_______________  What about \( cos \beta \)? \_____________. Is there any relationship between \( \alpha \) and \( \beta \)?

Property(of cofunctions).
A trig. function of an angle is always equal to its cofunction of its complementary angle.

ex. Special angles(acute): \( 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ \)

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>0(^\circ)</th>
<th>30(^\circ)</th>
<th>45(^\circ)</th>
<th>60(^\circ)</th>
<th>90(^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin ( \theta )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cos ( \theta )</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Find each of the following values.

ex. 1)( \( sin 30^\circ + cos 30^\circ \)^2 \) = \_______________  \hspace{2cm} 2) \( sin^3 30^\circ \) = \_______________

ex. \( sin 20^\circ = p \), without a calculator find \( sin 70^\circ \) = \____________  \hspace{1cm} \( cos 20^\circ \) = \____________

\( tan 90^\circ \) = \____________\hspace{2cm} \( sec 45^\circ \) = \____________

A tree casts a 30-foot shadow. The boundary line between the sunlight and the shade in midair makes an angle of 60\(^\circ\) with the ground. How tall is the tree to the nearest foot?
**Scientific notation:**

decimal expression of a number in the form \( n \cdot 10^m \), where \( 1 \leq n \leq 9 \)

**Conversion:**

a) To scientific notation:

\[
0.0042 = \underline{\hspace{5cm}} \quad 256.1 = \underline{\hspace{5cm}}
\]

b) To standard form:

\[
2.1 \times 10^{-4} = \underline{\hspace{5cm}} \quad 1.02 \times 10^3 = \underline{\hspace{5cm}}
\]

**Accuracy:**

We say 342 has a three figure accuracy, we say 3004 has a four figure accuracy

What about these numbers

\[
3000 \Rightarrow \underline{\hspace{5cm}}.
\]

In scientific notation: \(3 \times 10^3 \Rightarrow \underline{\hspace{5cm}}\) \(3.0 \times 10^3 \Rightarrow \underline{\hspace{5cm}}\) \(3.00 \times 10^{32} \Rightarrow \underline{\hspace{5cm}}\)

With degrees we say

<table>
<thead>
<tr>
<th>Significant digits</th>
<th>accuracy of sides</th>
<th>example</th>
<th>accuracy of angles</th>
<th>example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two significant digits</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three significant digits</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four significant digits</td>
<td></td>
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</tbody>
</table>
Solving Right Triangles - finding all of the sides and all of the angles.

We can solve a right triangle if we are given

1) Two of the sides  or  2) one of the sides and one acute angle.

Find angle A in triangle ABC with altitude CD of length h and angle B of measure 45°.

In the following figure

If C = 26°, r = 20, find x
Use the given two triangles drawn at 90° to each other. If ∠ABD = 27°, ∠C = 62°, and BC = 42, find x and then find h.

Applications.

Angle of elevation and angle of depression –

Bearing of a line L - acute angle formed by the north-south line and the line L.

Heading: begin from the north line in a clockwise direction (up 360°)
Examples

1. #2 page 91  an equilateral triangle has an altitude of 4.3 inches. Find the length of the sides.
   Can you construct a formula for the side of a general equilateral triangles with altitude $h$?

2. (6/92) A road up a hill makes an angle of $5.1^\circ$ with the horizontal. If the road from the bottom of the hill to the top of the hill is 2.5 miles long, how high is the hill.

3. (11/92) A person standing 150 cm from a mirror notices that the angle of depression from his eyes to the bottom of the mirror is $12^\circ$, while his angle of elevation to the top of the mirror is $11^\circ$. Find the vertical dimension of the mirror.

4. (#16/93) A man wandering in the desert walks 2.3 miles in the direction S $31^\circ$ W. He then turns $90^\circ$ and walks 3.5 miles in the direction N $59^\circ$ W. At that time, how far is he from his starting point, and what is his bearing from his starting point.

ex.4 –
Find the distance between Stacey and Amy using the given topographic map on page 86.
Use the fact that the distance between Amy and Stacey on the map is $3/8"$. 
ex. Find the height of the flagpole if you have the capability to measure angles. See example 7 page 89.

ex. Look at example 8 page 90.
**Vectors**

magnitude and direction:

- force, velocity, acceleration

Scalars: have only magnitude

**Def. Equality:**

Define addition of vectors:

Define Subtraction of vectors.

heading: begin from the north line and move an angle $\theta$ that is measured clockwise

air speed: speed before wind acts on object     ground speed: speed after wind acts on object

Components of a vector: a horizontal and a vertical vector can be added to end to obtain the original vector

**Examples:**

1. (14/108) A boat is crossing a river that runs due north. The heading of the boat is due east, and it is moving through the water at 12.0 mph. If the current of the river is a constant 3.25 mph, find the true course of the boat.

2. (18/108) A plane headed due east is moving through the air at a constant 180 mph. Its true course, however, is 65.0°. If the wind currents are moving due north at a constant rate, find the speed of these currents.
3. (22/108) A ship is 2.8° off course. If the ship is traveling at 14mph, how far off course will it be after 2 hours?

4. (34/109) A bullet is fired into the air with an initial velocity of 1800 ft/sec at an angle of 60° from the horizontal. Find the horizontal and vertical vectors of the velocity.
1. Draw a right triangle. Label the three vertex angles (vertices) and the three sides. Use your labeling to state the Pythagorean Thm.

2. Through how many degrees does the minute hand move in 15 minutes? __________
   In 60 minutes? ________

3. A trig. angle is said to be positive if it is measured in what direction? ___________________

4. What is the measure of an obtuse angle? __________

5. Complementary angles are ACUTE angles whose measures add up to? ________________

6. If two triangles have corresponding pairs of angles equal to each other, we say the triangles are ______________

7. Find the missing side.

8. Complete the following triangle

9. Complete the following triangle

10. A ladder is to reach a window on the third floor. If the window is 24 feet above the ground and for safety reasons the angle created by the ladder and the ground can not exceed 30°, then what is the shortest ladder that can be used so that the top of the ladder reaches the window?
1. Use the letter x, y, and r to define
   a) \( \sin \theta = \) __________
   b) \( \tan \theta = \) _________
   c) \( \sec \theta = \) __________

2. Find two angles that are coterminal to \( \theta = 40^\circ \) and their absolute value is less than \( 400^\circ \).
   ___________________   _______________

3. A forty-foot ladder leans against a wall. It touches the a point on the wall that is 20 feet above ground level. What angle does the ladder make with the wall?

4. Find the missing side of the following triangle.

5. Identify as a function or just a relation.

   a) 

   b) 

   c) 

6. In what quadrant are both \( x/y \) and \( y/r \) positive? ______________
1. Draw an angle θ. Label the initial and terminal sides.

2. Which of these angles represent a positive measure angle? ________________

3. Find an angle that is supplementary to 80° ________________

4. If α and β are complementary, then find the sum of α and β. ______________

5. Draw a right triangle. Label the vertices (ABC) and the sides (abc). State Pythagorean Thm.

6. Find the missing sides of each of the following right triangles.

   a. 
   b. 

8. Find the sides of each of the following triangles.

   a. 
   b. 

10. Find the measure of angle β.
1. In what quadrant or quadrants is
   a) \( x/y > 0 \) \( \rightarrow \) ______________
   b) \( x/r < 0 \) \( \rightarrow \) ______________

2. Given the points \( P(2, 5) \) find the radius vector (length) \( r \) from the origin to \( P \). \( \rightarrow \) ____________

3. Find the distance between
   a) \( A(0, -4) \) and \( B (-3, -4) \) \( \rightarrow \) ______________
   b) \( P(3, 3) \) and \( Q(-1, 6) \) \( \rightarrow \) ______________

4. Give me a rough sketch of each of the following curves.
   a) \( y = 2x + 4 \) by finding the \( x \) and \( y \) intercepts.

5. What quadrant is the angle \( -300^\circ \) in? \( -300^\circ \) is a quadrant __________ angle.

6. Draw the angle \( 400^\circ \) in standard position.
1. Find an angle that is complementary to $60^\circ$ ______________

2. Give me an angle $\theta$ so that $|\theta| < 360^\circ$ and $\theta$ is coterminal with $240^\circ$. ______________

3. What is the reciprocal function of $\sin \theta$? __________
   Write two reciprocal functions as an equation. $\sin \theta = __________$

4. Complete the following right triangle.

5. A wire is attached from the top of a 60 foot tower to the ground. The wire is grounded 60 feet from the foot of the tower. What is the length of the wire to the nearest foot? __________

6. Define $\cos \theta$ in terms of $x$, $y$, and $r$.

7. If $P(12, -5)$ is a point on the terminal side of $\theta$, then find $\sin \theta$ ______________

8. If the angle $-240^\circ$ is in standard form, then what quadrant is $-240^\circ$ in? ______________

9. Find a point on the terminal side of $90^\circ$. __________

10. The sun rises at 6:30 AM and sets at 6:30 PM. Assume the path to be part of a circle, through how many degrees does the sun move in 1 hour with respect to the earth. __________
1. State the definition of $\cos \theta$ in terms of $x$, $y$, and $r$. $\cos \theta =$ _______

2. Use the words opp., adj., and hypotenuse to define
   a) $\sin \theta =$ __________
   b) $\cot \theta =$ __________

3. State the three reciprocal identities in the proper form.
   a) $\csc \theta =$ __________
   b) $\sec \theta =$ __________
   c) ___________ $=$ ________

4. State the two ratio identities in the proper form.
   a) $\tan \theta =$ __________
   b) $\cot \theta =$ __________

5. State the three Pythagorean identities.
   a) $\sin^2 \theta +$ __________ $=$ __________
   b) 1 + __________ $=$ __________
   c) ___________ + ___________ $=$ $\csc^2 \theta$

HW: Day 4 – Jan. 23, 2002
January 23: page 35: 9, 15, 27, 30, 31, 35, 39, 41, 43, 45, 47, 51, 61, 63, 66, 71
January 25: page 43: 1, 6, 7, 9, 13, 15, 18, 20, 23, 26, 27, 35, 41, 45,
1. Find an angle that is complementary to $40^\circ$. _______________

2. If $\alpha$ and $\beta$ are supplementary, what is $\alpha + \beta$? ________

3. Find the missing side of each of the given triangle

4. A ladder leans against a building. The ladder is 30 feet long and it makes an angle of $60^\circ$ with the ground. How far is the bottom of the ladder from the building (bottom)? ______________

5. A person is running at 5 miles per hour. What distance will he travel in 24 minutes? ______________

6. True or False.

   a) Congruent triangles can be of different sizes
   b) Similar triangles can be the exact same size.

7. Which of these angles has a positive measure?

8. Plot the points $A(3, -2)$, $B(0, -4)$ and label the quadrants.

9. Find the quadrant or quadrants in which

   a) $x/r$ is positive. ____________
   b) $y/r$ is negative ____________
   c) $x/y$ is positive. ________
1. Which of these three relations also represents a function?

2. Draw the following circle \( x^2 + y^2 = 4 \)

3. Write the definition of \( \sec \theta \) in terms of opposite, adjacent, and hypotenuse.

4. Write the reciprocal identity for \( \csc \theta \).

5. Write the ratio identity for \( \tan \theta \).

6. What does \( \cos^2 \alpha + \sin^2 \alpha = \)?

7. If \( \alpha \) and \( \beta \) are complementary and \( \sin \alpha = 3/5 \), then find
   a) \( \csc \alpha \).
   b) \( \cos \beta \).
   c) \( \cos \alpha \).

8. A bird flies off the ground onto a 40 foot tree that is 60 feet away. If it flew in a straight path, what angle does its flight line make with the ground?
9. Reduce each of the following.

a) \( (\sin^2 \theta + \sin \theta) / (1 + \sin \theta) = \)

b) \( (\sin^2 \theta - \cos^2 \theta) / (\sin \theta + \cos \theta) \)

10. Prove or Disprove.

a) \( \csc \theta \sec \theta \tan \theta = \sec^2 \theta \)

b) \( \cos \theta - \sec \theta = -\sin^2 \theta / \cos \theta \)
1. Write the definition of $\sec \theta$ __________

2. What is the cofunction of $\cos \theta$ ______________

3. What is the reciprocal function of $\sec \theta$ ______________

4. Draw $\theta = -20^\circ$ in standard position

5. Complete the sides of the following two triangles
   a. b.

6. Write the three Pythagorean identities.

   ______________________

   ______________________

7. Write the two ratio identities.

   ______________________

8. From the top of a forty foot building a student spots a tree. The angle of depression from the building to the top of the tree is $70^\circ$. The angle of depression from the top of the building to the bottom of the tree is $74^\circ$. How tall is the tree? Assume the tree is located 40 feet from the building.

5. Prove

   $\sec \theta - \cos \theta = \sin^2 \theta \sec \theta$
Math 1303 – TI review – some possible type of questions

1. An angle $A$ is said to be in __________________________ provided its vertex is at the origin and its initial side is on the x-axis.

2. A positive measure angle is an angle that:

3. Two angles $A$ and $B$ are said to be __________________ provided ___________________________________________________

4. Two acute angles whose sum is 90° are said to be __________________

5. Supplementary angles are ________________________________

6. Two numbers $p$ and $q$ are said to be ____________________ provided $pq = 1$

7. The reciprocal function of $\sec \theta$ : __________________

8. Write a reciprocal identity for $\cot \theta = \______________$

9. State Pythagorean Theorem:

10. Complete the following triangle:

11. State the ratio identity for $\cot \theta = \______________$

12. State two of the Pythagorean identities:

13. Find the distance between the points $A (4, 0)$ and $B (7, -5)$

14. Find the $\sin \theta$ if $P (5, 8)$ is on the terminal side of $\theta$

15. Write the definition of $\sin \theta$ in terms of $x, y, r$ Write the definition of $\cot \theta$ in terms of opp., adj., hyp.

16. In what quadrant is $\sin \theta < 0$

17. If $\cos \theta = -5/13$ and $\theta$ is not in quadrant II, then find

\[
\sec \theta = \______________ \quad \sin \theta = \______________
\]
18. Find each of the following products in simplest form.
   a) \((\cos^2 \theta + \sin \theta)^2 = \) \______________
   b) \((\sin \theta - \cos \theta)(\sin \theta + \cos \theta) = \) \______________

19. Find \(\sin 90^\circ = \) \______________
    \(\cot 180^\circ = \) \______________

20. Sketch the graph of \(y = 4x - 8\)

21. What are the x and y intercepts of
   a) \(x = 2y + 2\) \______________ \______________
   b) \(y = x^2 - x\) \______________ \______________

22. What are the y-intercepts of \(x^2 + y = 3\) \______________

23. The roof of a house is to extend up 13.5 feet above the ceiling, which is 36 feet across. Find the length of one side of the roof.

24. 4 revolutions per minute = \______________ degrees per minute
    \(720^\circ = \) \______________ revolutions.

25. A person that moves at 9 feet per sec is moving at \______________ per yard

26. Prove each of the following identities.
   a) \(\csc \theta \cos \theta = \sec \theta\)
   b) \(\sec \theta \cot \theta - \sin \theta = \cos^2 \theta / \sin \theta\)
   c) \(\cos \theta (\sec \theta - \cos \theta) = \sin^2 \theta\)

27. Identify as a function or just a relation
   a) \(y = x^2\)
   b) \(R = \{(2, 3), (3, 4), (4, 5), \ldots\}\)
   c) \(x^2 + y^2 = 1\)
   d) \(\)