Math 130A – Math Fundamentals I
Observation-participation → success

Sets:
N = { 1, 2, 3, 4, ... } is called the set of ________________ or the set of _________________

When we “add” zero to this set, we get the following set;
W = { 0, 1, 2, 3, ... } and this set is called the set of _________________

Special sets:
We can talk about even numbers, odd numbers. The following would be called the set of
{ 1, 3, 5, 7, ..... } _________________ Describe the set of even numbers. _________________

What would be the smallest even number? ______

Later on we will talk about another set called the set of prime numbers. What is the smallest prime number?
set of prime numbers: { ____________________________ ... }

Back to whole numbers
Every whole number can be mapped (plotted) on a line.

------------------------------------------------->

We call this a number line. Notice that when two numbers are compared to each other the rightmost number is always larger. We can use the following symbols to compare two numbers
Reading from left to right;

> means: __________________________________ < means: __________________________________

≥ means: __________________________________ ≤ means: __________________________________

example: which of these statements is true

2 < 4 , 4 < 1  5 < 6  7 ≤ 7  0 > 3  4 ≥ 4

A number can be written in standard form

234 or 4,567 or 123,234

each of the digits has a place value beginning with the right-most digit:

one’s, ten’s, hundred’s, _____________, ______________, ________________ ......

How would you say

1,040: ___________________________  204: ___________________________

23, 356: ___________________________  3,001,230: ___________________________
Sometimes it may be of help if you write a number in expanded form:

\[ 23 = 20 + 3, \quad 345 = 300 + 40 + 5 \]
\[ 23,456 = 20000 + 3000 + 400 + 50 + 6 \]

Write in expanded form

304 = ___________________________  
1045 = ___________________________

**Rounding**

is sometimes used to approximate values and to give us an idea of the result so that we have an idea of what to expect when we use a calculator.

ex. round 125 to the nearest ten : ____________  
round 256713 to nearest ten-thousand ____________

ex. round 350242341 to nearest million ____________

Note: the digit to the right of the digit to be rounded determines whether to round up or down. If the digit is 5 or larger, round up – otherwise round down.

**Graphs are useful to represent statistical data.**

We can use

- pictographs: use symbols to represent quantities
- circle graphs: pie charts – represent and compare different quantities
- bar graphs, double bar graphs, line graphs, and double line graphs. (see some examples of these beginning on page 9)

Look at examples on page 14: # 90, 96, 99, 102, 103

HW: page 13

1, 8, 14, 20, 34, 36, 44, 52, 58, 70, 76, 83, 86, 97, 100, 107
Addition of Whole numbers

3 + 5 = 8  
3 and 5 are called the addends, 8 is called the sum

What is the sum of 4 and 8? __________

Other phrases that indicate addition:

- added to: 4 is added to 8 __________
- more than: 4 more than 8 __________
- the sum of 4 and 8 __________
- 8 increased by 4 __________
- the total of 8 and 4 __________
- 4 plus 8 __________

All the statements above can be written as _4 + 8_ and as you can probably agree _8 + 4_. Why?

ex. _During the last three years:_ 50 students have made A’s in math 130A, 88 have made B’s, 150 have made C’s, 90 have made D’s, and 84 have made F’s or dropped the class.

How many students were there during the last three years? __________

How many of them passed successfully (able to move onto math 130B)? __________

ex. _The following picture_ illustrates the comparison of male to female students that smoked during the last five years. Which year produced the largest number of smokers?

Which year produced the largest number of smokers? __________

ex. _A family’s budget can be separated into the following:_

30 % goes towards utilities  
20 % towards food and other essentials  
35 % towards credit bills

the remaining amount goes into the savings account.

Draw a pie chart (circle graph) and determine what percent is spent (not saved).

spent: __________ %

saved: __________ %
ex. A student knows that it will cost $17843 to attend an out of state university. In addition, food and minor essentials will add up to an additional $11431. Car expenses plus other expenses will total $8190. Estimate the total cost.

Estimate the cost to the nearest hundred. _________

When we use a letter to represent a number, we call the letter a variable. If we use a given value for that letter, we can evaluate an expression.

ex. Find c + 349 when c = 874 (Evaluate) _________

ex. Find 497 + F when F = 398 (Evaluate) _________

Notice: Does 3 + 2 = 2 + 3 ?  5 + 4 = 4 + 5 ? ..... Why or Why not ?

More estimate:

\[
\begin{array}{cccc}
3968 & & & 309 \\
+ & 853 & + & 12 \\
\hline
\end{array}
\]
Properties of Whole Numbers:

1. The addition Property of Zero:
The sum of a number and zero is the number: \( c + 0 = c \) or \( 0 + c = c \)

   ex. What is the result of \( 5 + 0 = \) _____ \( 0 + 312 = \) ________ \( r + 0 = \) _____

2. The Commutative Property of Addition:
two numbers can be added in either order resulting in the same value: \( a + b = b + a \)

   Ex. Write using the commutative property and find the total
   \( 17 + 98 = \) ________ \( 213 + 197 = \) _______

3. The associative property of Addition:
When adding three numbers, the numbers can be grouped in “any” order; the sum will be the same

   \[ (a + b) + c = a + (b + c) \]

   ex. Find \( (312 + 589) + 11 = \) ______________________

   Rewrite using the associative law and then find the result.

   Note: Which law is being used \( \rightarrow \) \( (3 + 2) + 5 = (2 + 3) + 5 ? \) ________________

Other examples:
ex. Write the missing side using the commutative property of addition. \( 4 + a = \) ____________

ex. Find \( x + 0 = \) __________

ex. Which property is being used \( 3 + (x + y) = (3 + x) + y \) ________________

An equation is a numerical expression with the equals sign

\( 3 + 5 = 8 \) , or \( x + 2 = 4 \) , ... are examples of equations but \( x + 3 + 5 \) is not an equation. Why?

An equation can be true or false.
\( 4 + x = 10 \), this is true if \( x = 6 \) but it is false if \( x = 3 \).

When the equation is true for a particular value, we call this value a solution of the equation.

Ex. Find the solution of each equation

1. Find a if \( 1 + 2a = 9 \) \( \rightarrow \) a = _____

2. Is 270 as solution of \( x + 213 = 583 \) \( \rightarrow \) x = ________

3. Is 324 a solution of \( 430 + x = 106 \) \( \rightarrow \) x = ________


Other Examples:
A sample of 100 students was taken. 60 were enrolled in a math 130A class 20 were in Math 130B, 10 were in a Business Math Class, 5 were in Math(science) class, and the remaining students were not in any math class. Construct a circle graph (pie chart to describe the setting.

How many students were not in a developmental class?

Which group was the largest?

The graph above (bar graph) represents the number of A’s in a math class over the past four years.

What year had the most A’s? What two consecutive years had the largest change in the number of A’s?

What is the total number of A’s in the four-year period?

Ex. The double bar graph below is a fictional three-year study of male-female smokers. In what year were the number of female and male smokers closest to each other?

What year had the largest difference?
ex. Find a solution of $x + 5 = 12$, $x = \phantom{10}$

Find a solution of $2 + x = 10$, $x = \phantom{10}$

ex.

The circle graph represents the total number of students from each area.

- 370 From West Texas
- 215 From Central Texas
- 75 From East Texas
- 42 from outside of Texas

What is the total number of students? _____

What is the total number of students from Texas? ____

Estimate the following sums:

- $345 + 492 \Rightarrow \phantom{1000}$
- $9765 + 8765 \Rightarrow \phantom{10000}$

What kind of number do you get when you add two natural numbers? __________

What about the sum of two whole numbers? __________

What about the sum of two odd numbers? __________

What about two prime numbers? __________

Conclusion?
Subtraction:

12 - 4 = 8:  12: minuend,  4: subtrahend,  8: difference

examples:

345 - 243 = _____________

\[
\begin{array}{c}
3604 \\
-2728 \\
\hline
\end{array}
\]

345 - 243 = 102

Other phrases that have the same meaning as subtraction.

10 minus 4, 10 less 4, 4 less than 10, the difference between 10 and 4, 10 decreased by 4

They can all be represented by 10 – 4

Evaluate the following: b - a, when b = 357 and a = 288

Is 42 a solution of x - 12 = 52

Is 24 a solution of 60 - x = 36

Estimate the difference between 480 and 270

We talked about the commutative and associative properties with addition. Do these properties hold with subtraction?

In other words:

Is a – b = b – a?

Is (a – b) – c = a – (b – c)?

We said we can add two whole numbers and get a whole number. We can add any two even numbers and get an even number. This mean the addition property is closed in each of these sets.

Is addition closed in the set of odd whole numbers?

What about in the set of prime numbers?

Natural numbers?

Whole numbers?
More examples:
Some introduction to geometry.

point: \( P \)

line:

ray:

line segment:

Angle: measured in degrees

right angle, rays are perpendicular

Planes: think of flat surface; floor, board, wall,...

lines on a plane can be

parallel

perpendicular

intersecting

Polygon: closed figure on a plane determined by three or more line segments

triangles, quadrilaterals, pentagon, ....
Quadrilaterals:

- **rectangles:**
  - 12 inches
  - 4 inches

- **squares:**
  - 4 inches
  - 4 inches

- **parallelograms:**
  - 3 cm
  - 15 cm

End of day 2 (Spring 2005)

**Perimeter:** of a plane figure: → the distance around

Find the perimeter of the figures above.

HW: page 33

6, 13, 20, 22, 24, 29, 40, 46, 48, 52, 55, 60, 64, 72, 84, 86, 93, 100, 107, 112, 115, 122, 124, 126, 131, 136, 138, 140, 141, 146
Multiplication

Multiplication is repeated addition. The numbers to be multiplied are called factors and the result is called a product.

ex. \( 4 + 4 + 4 + 4 + 4 = 5 \) fours added together, which we can write as \( 4 \cdot 5 \).

ex. Six different classrooms are being used during this time period. Each room has a capacity of 42 students. What is the total capacity in these six rooms?

\[ \text{Total capacity} = 6 \times 42 \]

ex. A shipment of 46 boxes of light bulbs arrive at a store. Each box contains 25 bulbs. How many light bulbs are available?

\[ \text{Total light bulbs} = 46 \times 25 \]

ex. A store sells packs of 245 firecrackers. How many fire crackers do they have in total if there are 458 packs in the store?

\[ \text{Total firecrackers} = 245 \times 458 \]

Notation: Different ways to represent multiplication: \( 6 \times 8, 6(8), (6)(8), 6a, 6(a) \)

These all mean a product of two numbers. Each of these numbers is called a factor and the result is called the product.

Examples:

Find the product of 6 and 8. \[ 6 \times 8 = \]

Find the product of 12 and 43. \[ 12 \times 43 = \]

Find \( 345 \times 567 = \)

What is the largest factor in the expression \( 12(4) = 48 \)? \[ \text{ Factor } = 4 \]

Estimate the following product. \( 34 \times 42 \rightarrow \)

Find the solution of each of the following. \( 2x = 24 \rightarrow \)

\( 6(x) = 18 \rightarrow \)

Phrases that mean the same as multiplication:

product of 8 and 10, 8 multiplied by 10, twice(three,... as large as 20, …
Properties of multiplication

1. \( a \cdot 0 = 0 \cdot a = 0 \) prop. of zero
2. \( a \cdot 1 = a = 1 \cdot a = a \) prop. of 1
3. \( ab = ba \) commutative law of multiplication
4. associative law of multiplication \( (ab)c = a(bc) \)

Find the value of \( a \) if \( 5 \cdot a = 0 \)

\[ 5a = 5 \rightarrow \]

Is this an example of the commutative law: \( 3(2 \cdot 5) = 3 (2 \cdot 5) \)?

Is \( 2 \) a solution of \( 2x = 4 \)? Why?

Notice: some products of large numbers are actually easy to perform.

\[ 3 \times 40 = \] _________
\[ 60 \times 40 = \] _________
\[ 300 \times 60 = \] _________

Exponents

Just like multiplication is repeated addition, exponents deal with repeated multiplication.

ex. \( 3^2 = 3 \cdot 3 \) = _________
ex. \( 4^3 = 4 \cdot 4 \cdot 4 \) = _________

In the notation \( 4^5 \), 4 is called the base and 5 is called the power or the exponent.

ex. \( 3^4 = \) _________
ex. Find the square of 7 _________
ex. Find the cube of 5 _________

When we write \( 4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \), \( 4^5 \) is called the exponential notation (form) the right side is called the expanded form.

Note: of special interest

\( a^0 \) is defined to be 1, if \( a \neq 0 \). In the event that \( a = 0 \), we say \( 0^0 \) is undefined.

ex. \( 5^0 = \) _________
ex. \( 4^3 = \) _________
ex. \( 20^4 = \) _________

Note2: \( 10^n = \) one followed by n zeroes

ex. \( 10^5 = \) _________
ex. \( 10^8 = \) _________
Examples:

1. Evaluate:
   a) \(2^2 \cdot 3^3 = \) \[\text{__________}\]
   b) \(2^4 \cdot 4^2 = \) \[\text{__________}\]
   
   c) \(x^2y\) when \(x = 2\) and \(y = 3\) \[\rightarrow \text{__________}\]

   d) \(x^3y^2\) when \(x = 2\) and \(y = 4\) \[\rightarrow \text{__________}\]

   e) \(10^4 = \) \[\text{__________}\]

   f) \(10^{10} = \) \[\text{__________}\]

2. Find \(4,538 - 2929\). \[\rightarrow \text{__________}\]

3. Is \(x = 24\) a solution of the equation \(29 = 53 - y\)? \[\rightarrow \text{__________}\]

4. Review some of the Geometry on page 30 of your text.
   What is the perimeter of a piece of land that is in the shape of a triangle with sides
   2000 ft by 439 ft by 1827? \[\rightarrow \text{__________}\]

5. A rectangular yard is to be fenced. If the yard is 150 feet long by 40 feet wide, what is the
   perimeter of the yard? \[\rightarrow \text{__________}\]

6. It costs $4 per yard to border a rectangular yard that measures 15 yards by 21 yards. What
   will it cost to border this yard?

7. Which one has a larger perimeter
   a rhombus with side of length 6 feet or an equilateral triangle of length 8 feet?
Division

\[ \frac{24}{6} = 4 : \] dividend, divisor, quotient

Notation: \[ 24 \div 6 = 4, \quad 24 / 6 = 4, \quad 6\sqrt{24} \]

Phrases: find the quotient of 30 and 6 \[ \rightarrow \] ____________

40 divided by 8 \[ \rightarrow \] ____________

Find 3624 \div 8. ____________

Notice that there is a connection between division and multiplication: \[ 24 \div 6 = 4, \] since \[ 4 \times 6 = 24 \]

If we use this connection, some of the properties below make more sense.

Properties:
If \( a \) is not zero, then

1. \( 0 \div a = 0 \)
2. \( a \div a = 1 \),
3. \( a \div 1 = a \),
4. \( a \div 0 \) is undefined

Is division commutative? Is it associative?

ex. A sack contains 126 balls. These balls are to be placed in sets of six in smaller boxes. How many boxes can be filled? ____________

What if you had 304 balls? ____________ How many boxes will be needed? ____________

ex. A basketball tournament is played with four players per team. How many teams can be formed with exactly 68 players?

ex. Other examples:

\[ 4824 \div 12 = \quad \text{__________} \quad 398 \div 35 = \quad \text{__________} \]

ex. Estimate: \[ 29435 \div 501 \]

ex. Evaluate: \( x \div y \) when \( x = 462 \) and \( y = 22 \)

ex. Is \( y = 12 \) a solution of \( y / 4 = 3 \)
Some divisions can be done rather easier than it looks.

\[
246 \div 12 \rightarrow \frac{246}{12} = \frac{123}{6} \rightarrow 123 \div 6 = ________
\]

\[
a^3 \div a^2 = ? \quad \text{We know that } a^3 = a \cdot a \cdot a \quad \text{and} \quad a^2 = a \cdot a. \quad \text{So}, \quad a^3 \div a^2 = \frac{a^3}{a^2} = \frac{a \cdot a \cdot a}{a \cdot a} = ? \quad ________
\]

\[
300 \div 60 = ________ \quad 480 \div 40 = ________ \quad 3600 \div 60 = ________
\]

Notice that in some division problems: there is a remainder different that zero.

\[
32 \div 7 = ? \quad 307 \div 100 = ________
\]

In some cases the remainder is zero and those kind of divisors have a special property. They are called divisors(factors) of the dividend.

\[
24 \div 6 = 4 \text{ with no remainder}. \quad \text{We say 4 and 6 are divisors of 24 (factors)}
\]

Closure:

Which of the following sets are closed under the given operations.

<table>
<thead>
<tr>
<th></th>
<th>addition</th>
<th>multiplication</th>
<th>subtraction</th>
<th>division</th>
</tr>
</thead>
<tbody>
<tr>
<td>natural numbers</td>
<td></td>
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<tr>
<td>whole numbers</td>
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<tr>
<td>integers</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
FACTORS

We can write any natural number as a product of other natural numbers.

ex. $8 = 4 \times 2$, $1 \times 8$  

ex. $12 =$

These numbers are called factors (divisors) of 8 and 12 respectively.

When we look at all of the divisors of 8, we have 1, 2, 4, 8. The divisors different from 8 are called proper divisors.

What are the proper divisors of 6? ________________  
What are the proper divisors of 16? ________________

**Def. (definition)** An abundant number is a natural number whose proper divisors add up to more than the number itself:

12: 1, 2, 3, 4, 6, sum $> 12$

There are 246 abundant numbers between 1 and 1000 (for example; 18, 20, 24, 30 )

For example: is 40 an abundant number? ____________________________

**Def. A perfect number** is one whose sum of proper divisors equals the number

Here are a few perfect numbers: 6, 28, 496

Why is 16 not a perfect number? ____________________________

**Def. A prime number** is a natural number greater than 1 that has only 2 natural # factors (divisors); 1 and itself.

examples.

List all of the prime numbers between 20 and 30. __________________

Find the product of the first three prime numbers ________

Find the sum of the first four odd prime numbers. ________

**Def. A composite #:** a natural number that is not a prime number but it is greater than 1.

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Prime Factorization of a composite number

Note: (prime factorization) every composite number can be written as a product of other numbers and in particular as a product of prime numbers in exponential form.

When we write a composite number as a product of prime numbers we call this product the prime factorization of the number.

\[ \text{ex. } 9 = 3 \times 3 = 3^2 \quad \text{ex. } 12 = 2 \times 2 \times 3 = 2^2 \times 3 \]

Find the prime factorization of

\[ 20 = \quad \text{ex. } 160 = \quad 37 = \quad \]

Suppose I told you that I am thinking of a number whose prime factors are 7, 2, and 5. What’s the number? _______

Given that 3 and 7 are two of the prime factors of 1134. Could you find the other prime factors? ______________

ex. One oz. of cheese contains 115 calories. Find the number of calories in 4 oz. of cheese.

ex. An ex-football player rushed for 10842 yards during his career. If he carried the ball 2112 times, then estimate the number of yards per carry.

Expected end of day 3 (spring 2005)
Solving equations with whole numbers.

If we want to find the solution of \( x + 2 = 120 \) we can easily guess the solution of such an equation.

ex. What is the solution of \( 240 - x = 200 \) \( x = ________ \)

What about \( 2x = 200 \), \( x = __________ \)
And \( x/2 = 50 \), \( x = ___________ \)

Some equations are easier to solve than others.

ex. what is the solution of \( x = 12 \) \( x = ________ \)
ex. What is the solution of \( 2x = 24 \), \( x = ________ \)

When two equations have the same solution, we say the equations are equivalent.

But as the equations get more difficult we may need to use the next couple of properties to convert an equation to a simpler form that has the same solution, an equivalent equation.

Properties.

1. We can add (or subtract) the same quantity from both sides of an equation without changing the solution of an equation.

2. We can multiply (or divide) both sides of the equation by the same number without changing the solution.

Cancellation laws:

\[
\frac{c}{a} = \frac{ab}{a} = b , \quad \frac{42}{7} =
\]

\[
a - a = ______
\]

Although you can probably find the solution of each of the following equations without any work – use the properties above to solve the equations.

ex. \( 4 + x = 20 \) \( x = ________ \)
ex. \( x - 20 = 12 \) \( x = ________ \)

ex. \( x/2 = 6 \) \( x = ________ \)
ex. \( 3x = 21 \) \( x = ________ \)
ex. The sum of eleven and a number equals 65. Find the number.

ex. The product of 20 and a number is equal to 240. Find the number.

ex. Six more than a number is equal to 24. Find the number.

ex. Ten less than a number is equal to 20. What is the number?

HW: page 71 2, 10, 18, 24, 26, 32, 36, 38
Order of operations

Find the answer to

\[ 2 + 4 \cdot 2 - 6 = \_\_\_\_\_\_ \quad 4 - 6 \div 2 = \_\_\_\_\_\_ \quad 4^2 - 5 \cdot 2 = \_\_\_\_\_\_ \]

It matters what operation is performed first. **PEMDAS:** Please Excuse My Dear Aunt Sally.

1. Parentheses (grouping)
2. Exponents
3. Multiplication and division – left to right
4. Addition and Subtraction – left to right

ex. \[ 14 \cdot (3 + 2) \div 10 = \_\_\_\_\_\_ \quad \text{ex.} \quad 3^2 \cdot 2^2 + 3 \cdot 2 = \_\_\_\_\_\_ \]

ex. \[ 15 - (7 - 1) \div 3 = \_\_\_\_\_\_ \quad \text{ex.} \quad 3(5 + 3) \div 8 = \_\_\_\_\_\_ \]

ex. Evaluate when \( a = 2 \) and \( b = 4 \) \[ 4a - 3(b - a)^2 = \_\_\_\_\_\_ \]

HW: page 75 1, 5, 12, 20, 27, 32, 37,

Review Ideas:
Look on page 79(79-82) to see a complete outline of everything that has been covered up to this point(Chapter Summary)

Additional Work:
page 83 Chapter Review, page 85 Chapter test
Chapter 2
Introduction to Integers.

Our discussion so far has centered on these numbers:

\{ 1, 2, 3, 4, ... \} → which we call the set of ___________________ or the set of __________________

\{ 0, 1, 2, 3, 4, ... \} → which we call the set of ________________

We have already talked about whole numbers and the number line.

There are quantities that can not be represented by whole numbers – the first of these is introduced below.

For example when we say

“ I owe $200”,

“ My check balance is $20 below”

“ I lost $40 “

“the value of a stock has decreased by $2”

Phrases like these require a different kind of number. We introduce the negative integers.

The set \{ ... –4, -3, -2, -1, 0,  1, 2, 3, 4, ... \} form the set of integers.

Numbers greater than zero (to the right of zero) are called positive numbers (positive integers - in this case )

and numbers less than zero (to the left of zero) are referred to as negative number ( negative integers in this setting )

What about zero ? ______________

We can discuss the set of integers in the same way we discussed whole numbers. This is the pattern that we will continue to use throughout the set of numbers that we will study.

ex. What number is 4 units to the right of –6 ? _________

ex. What number is 9 units to the left of  6 ? __________

ex. Place the correct symbol ( < or > )

\[ 12 \] _______ 102 \[ -4 \] _______ 8 \[ -4 \] _______ 0 \[ -20 \] _______ - 41

We can also think of the integers as being a certain distance from the origin.

ex. 4 is four units from the origin to the right of zero. \quad ex. –5 is five units from the origin to the left of zero
Def. Two numbers that are the same distance from the origin but in opposite directions are called **opposites**

ex. What is the opposite of 12 ==> _______  of -34 ==> _______

What about the opposite of 0 ? _______

ex. $-x$ can be read as the opposite of $x$.

What is the opposite of $-2$ ? _______

What is the opposite of $-( -4 )$ ? ______

We can now talk about addition, subtraction, multiplication, and division of integers.

Instead of writing $4 + 6$, we think of $(+4) + (+6)$.

$12 - 8$, we think of $(+12) - (+8)$

These are operations we have done with whole numbers but now we are extending them to integers. We should expect no difference in the following examples.

ex. $(+4) + (+6) = _______

Notice that this problem can be written as $4 + 6$.

ex. $(+12) + (+8) = _______

ex. $(+12) - (+7) = _______

Notice that this problem can be written as $12 - 7$.

We know some of these results but because of the introduction of negative integers let’s write a couple of rules.

There are more signs to worry about -- we have to be more careful with the symbols + and -.

Def. The absolute value of a number is the distance from zero to the number. We use the symbol $| |$ to describe the absolute value of the number.

The absolute value is never negative.

ex. What is the absolute value of 10, Solution: $|10| = _______

What is the absolute value of $+15$? Here $+$ is used not as an addition symbol but as a plus sign to indicate 15 is positive. Solution: $|+15| = _______

What about the absolute value of $-2$? How far is $-2$ from the origin? Solution: $|-2| = _______

Notice the use of the - symbol can represent subtraction(minus) or a negative sign.

the use of the + symbol can represent addition or positive sign.

Properties. The absolute value of a positive number is _______

The absolute value of a negative number is _______

The absolute value of zero is _______
Other examples.

ex. A computer is bought for $1475. A printer costs $425 and an external ZIP drive sells for $65. The store will finance the whole package if you do not have sufficient money. You have $1920 available. Is this enough, if so what change will you get back. If not, then what amount will be financed?

change ______________  financed ______________

ex. The length of the side of a rectangle is 12 inches. If the perimeter of the rectangle is 40 inches, then what is the area of the rectangle.

More examples of integer representations:

Addition and Subtraction of Integers

Note: \{ 1, 2, 3, 4, ... \} is called the set of \__________\ but it is also called the set of \__________\
or in terms of integers: the set of \__________\

While \{ 0, 1, 2, 3, ... \} is called the set of \__________\ or in terms of integers: it can also be called the set of \__________\

Start with what we know; whole numbers.
ex. We noticed that addition of positive integers the subtraction of positive integers is nothing more than
the addition and subtraction of whole numbers.

\[(+51) + (+75) = \__________\ \quad (+73) - (+62) = \__________\]

Which without much more explanation leads us to the following rules of addition.

Recall that in the expression; \(2 + 5 = 7\), 2 and 5 are called \__________\ 

Properties of Addition:

1. To add integers with the same sign, add the absolute values of the numbers and then attach the sign of the addends (keep the same sign).
   - \(+12\) + \(+25\) = \__________\ 
   - \((-27\) + \((-42\) = \__________\ 
   - \((35\) + \((56\) = \__________\ 
   - \((-172\) + \((-321\) = \__________\ 

2. If the integers have different signs, then find the absolute values. Subtract the smaller from the largest and then attach the sign of the larger.
   - \((+42\) + \((-27\) = \__________\ 
   - \((-45\) + \((39\) = \__________\ 

Note: What happens when we add a number and its opposite? We call the opposite of a number its \underline{additive inverse}. 
   - Note this special result: the sum of a number and its opposite is always equal to \__________\ (this is called the additive identity)
   - \((+5\) + \((-5\) = \__________\ 
   - \((-567\) + \((567\) = \__________\ 

More examples:

Find
\[-256 + (-325) = \__________\ \quad (+3125) + (+354) = \__________\]
There are other properties of integers, some of which we are familiar from the whole number properties.

Properties:

1. The addition property of zero
   If \( c \) is an integer, then \( c + 0 = 0 + c = c \) ➔ What is 0 called? ________________

2. The Commutative property of Addition
   If \( a \) and \( b \) are integers, \( a + b = b + a \)

3. The associative Property of Addition
   If \( a \) and \( b \) and \( c \) are integers, then \( ( a + b ) + c = a + ( b + c ) \)

The next rule is different from the ones that we have seen in our study of whole numbers.

4. The Inverse property of Addition
   If \( c \) is any integer, then the sum of \( c \) and its opposite \( c + ( -c ) = (-c) + c = \) __________
   \( c \) and \((-c)\) are called additive inverses
   (they are also called? __________)

Examples:

Is -2 a solution of \( x + 4 = 2 \) ?
What is -124 increased by 90 ?

What is 12 decreased by 20 ?

You check balance is $42. What will the balance be after a check for $54 is written? (before any penalty is taken)

What is the additive inverse of \( c \)? __________

What number when \( x \) results in the additive identity? __________
Subtraction of integers.

Subtraction of integers is no different than adding integers as long as we keep the following operation in mind.

ex. 6 - 4 = ( +6 ) - ( +4 )
    Subtracting is the same thing as adding the opposite. So we write the above problem as ( +6 ) + ( -4 )
    So, 6 - 4 = ( +6 ) + ( -4 ) =

ex. - 12 - ( - 8 ) =

ex. - 20 - ( + 20 ) =

ex. - 20 - ( + 20 ) =

ex. Simplify. - 4 - 20 - ( - 12 ) - 4 - ( + 4 ) =

ex. Find the square of - 4.

ex. What is - 2 cubed?

ex. Evaluate when x = -2 and y = - 1
    x^2 - xy =

HW: page 111
4, 16, 23, 26, 36, 45, 54, 56, 59, 63, 68, 73, 78, 84, 89, 92, 97, 99,
104, 114, 121, 127, 132, 134, 140, 143, 145
Multiplication and Division of integers.

Multiplication

Rule 1:
To multiply integers with the same sign, multiply the absolute values of the factors. The product is positive. (the product of two numbers with the same sign will always be positive)

-3 • (-4) = _______  
(+3) • (+5) = _______

Rule 2:
To multiply integers with opposite signs, multiply the absolute values of the factors. The product will be negative. (the product of two numbers with opposite signs will always be negative.)

ex. Find the product of the numbers; 12, -13, and +15

ex. Let x = -4, y = +5, and z = 8. Find \( -x^2 - xy - yz \).

ex. Evaluate. \(-4x\) when \(x = -2\)

ex. is \(x = -2\) a solution of \(-2x = -4\)?

Once we introduce multiplication and addition – we can bring up another useful property of numbers.

Distributive Law:

\[a (b + c) = ab + ac\]

\[2(3 + x) = 2(3) + 2(x) = 6 + 2x\]

\[5(1 - x) = 5(1) - 5(x) = 5 - 5x\]

Use the distributive law to simplify:

a) \(4(3 + a) = \) ____________  
b) \(2(b + 3) + 4(b - 2) = \) ____________

Properties:

1. Multiplication property of zero: \(a \cdot 0 = 0 \cdot a = 0\)

2. Multiplication property of one: \(a \cdot 1 = 1 \cdot a = a\)

3. Commutative property of multiplication: \(ab = ba\)

4. Associative property of multiplication: \((a \cdot b) \cdot c = a \cdot (b \cdot c)\)
Division:

What about division? Is it much more different? Same idea applies because of the relation between division and multiplication.

ex. \( 8 \div 2 = 4 \) means \( 4 \cdot 2 = 8 \). The sign rules are similar (same).

**Rule 1:**
To divide two numbers with the same sign, divide the absolute values of the numbers and assign a positive sign to the result.

\[
( - 84 ) \div ( - 21 ) = __________ \quad 312 \div ( + 6 ) = __________
\]

**Rule 2:**
To divide two numbers with different signs, divide the absolute values of the numbers and assign a negative sign to the result.

ex. Find the quotient of \( 4893 \div ( - 21 ) \). \( \Rightarrow \) ________________

ex. Find \( ( - 24 ) \div ( - 4 ) \div ( 3 ) \div ( - 1 ) \). \( \Rightarrow \) __________

ex. Evaluate \(-a / b\), when \( a = -12 \) and \( b = -3 \)

ex. What is \( 145 \div 0 ? \) __________

**Note:** given a fraction, the fraction can be written in at least three different ways

\[
\frac{-a}{b} = \frac{a}{-b} = \frac{-a}{b},
\]

for example: write \( 12 \div ( - 4 ) \) in three different ways (as fractions)

ex. At the last weigh in the following five weights changes were recorded \(-5, -2, +3, -2, -4\)
What was the average change in weight?

HW: page 123
9, 24, 32, 39, 42, 45, 46, 52, 59, 62, 68, 75, 80, 86, 91, 92, 97, 100, 104, 108, 111, 115, 119, 122
As we did with whole numbers, we can solve equations. Now, our solutions may be integers instead of just whole numbers.

ex. Find the solution of \( s + 12 = 14 \), \( s = \) ____

But what about \( x + 12 = 2 \) ? \( x = \) ______

Or \( 2x = -12 \), \( x = \) ______

As before the two operations of equations can still be performed.

1. We can add equal quantities to both sides and still have the same solution. (same with subtraction)
2. You can multiply both sides by the same nonzero quantity and still have the same solution. (same with division)

\[ \text{ex. } 3x - x = 4(x + 1) \]
\[ \text{ex. } 2 - 2(x + 1) = x \]

Other examples:

The product of negative six and a number is negative forty-two. Find the number.

The US balance of trade in 1980 was $305,755 million more than the US balance of trade in 1999. What was the US balance of trade in 1999?

A small store sells watermelons from a local farmer. The shopkeeper wants to make a total profit of $130 from the sale of 50 of the melons. He bought them at a price of $120 for the whole bunch. What price should he sell each of them to make that profit?

HW: page 133
5, 10, 15, 20, 25, 30, 35, 40, 45, 48
Order Of Operations.  Please Excuse My Dear Aunt Sally

ex.  $-16 \div 2 + 8 = \underline{\text{___________}}$  

ex.  $4 \left( 2 - 7 \right) \div 5 = \underline{\text{___________}}$

ex.  $-27 - \left( -3 \right)^2 - 2 - 7 + 6 \cdot 3 = \underline{\text{___________}}$  

ex.  $\left( -2 \right)^2 - 5(3) - 1 = \underline{\text{___________}}$

Evaluate the variable expression given  $a = -2$,  $b = 4$,  $c = -1$,  and  $d = 3$

ex.  $6b \div \left( -a \right) = \underline{\text{___________}}$  

ex.  $\left( d - b \right) / c = \underline{\text{___________}}$

ex.  $\left( b + c \right)^2 + \left( a + c \right)^2 = \underline{\text{___________}}$  

ex.  evaluate $b \div ac. \underline{\text{___________}}$

HW: page 137
  2, 6, 10, 14, 18, 22, 30, 34, 40, 45, 53

Additional Work:
  page 143: chapter review  
  page 145: chapter test  
  page 147: cumulative review
1. Properties:
   Commutative Law of Addition: _______________  Commutative law of multiplication: _______________
   Distributive Law: _______________

2. Sets of Numbers
   Natural Numbers: _______________  Whole Numbers: _______________
   Prime Numbers: _______________  Integers: _______________
   Set of positive integers: _______________

3. Round:
   2349 (hundred): __________  5555 (thousand): __________

4. Estimate:
   $74 + 872 \rightarrow$ __________  $451 - 24 \rightarrow$ __________
   $52 \times 382 \rightarrow$ __________  $3262 \div 57 \rightarrow$ __________

5. Factors
   of 48 $\rightarrow$ _______________  Prime Factorization of 48: $48= $ _______________

6. Exponents:
   $3^0 = $ __________  $0^3 = $ __________  $0^0 = $ __________  $3^4 = $ __________

7. Evaluate if $x = -2$ and $y = -3$
   $(x - y)(x + y) = $ __________  $x - y(3 - x) = $ __________

8. Find
   $392 + 79 = $ __________  $493 - 258 = $ __________  $342 \times 32 = $ __________  $357 \div 22 = $ __________

9. Simplify
   $| -2 | - | 6 | = $ __________  $- | - 4 | = $ __________  $- (3 - 5) = $ __________
   $3 - 4(4 - 7) = $ __________  $2^2 + 8 \div 4 - 2 = $ __________  $12 \times 3 + 8 \div 2 - 2 = $ __________

10. Find the solution of
    $2x = 28, x = $ __________  $x/2 = 6 \rightarrow x = $ _______.  $x - 4 = -9 \rightarrow x = $ _______,  $x + 4 = 3, x = $ _______
Ch. 3 Fractions

{ 1, 2, 3, … }  ➝  { 0, 1, 2, 3, … }
{ …. − 2, -1, 0, 1, 2, … }  ➝

Set of integers are closed under addition, subtraction, and multiplication. What about division?

**LCM: least common multiple**

Multiples of a number: the product of that number and the natural numbers.

multiples of 3:  3•1 = __,  3•2 = __,  3•3 = ____  ....  ➝  3, 6, 9, ...

multiples of 7:

Common Multiple: multiple of two or more numbers is a common multiple of those numbers.

Find a common multiple of  6 and 8 ➝

Find a common multiple of  4 and 9 ➝

**LCM:** the smallest of all the common multiples of the given numbers is called the least common multiple ( LCM)

Find the LCM of  6 and 12 ➝

Find the LCM of 12 and 16 ➝

Find the LCM of 20 and 36 ➝
Using the prime factorization of a number:
Find the LCM of
1) write the prime factorization of each number  
2) circle the highest power of each number  
3) take the product of the circled numbers

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Prime Factorization</th>
<th>LCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 and 12</td>
<td>( 6 = 2 \times 3 )  ( 12 = 2^2 \times 3 )</td>
<td>( 2^2 \times 3 = 12 )</td>
</tr>
<tr>
<td>12 and 16</td>
<td>( 12 = 2^2 \times 3 )  ( 16 = 2^4 )</td>
<td>( 2^4 \times 3 = 48 )</td>
</tr>
<tr>
<td>20 and 36</td>
<td>( 20 = 2^2 \times 5 )  ( 36 = 2^2 \times 3^2 )</td>
<td>( 2^2 \times 3 = 12 )</td>
</tr>
</tbody>
</table>

GCF: Greatest Common Factor
factor: a number that divides a given number is called a factor of that number

write the factors (divisors) of 48 \( 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 \)
Note: proper divisor – any of the divisors except for the number itself

ex. the factors of 4 are: 1, 2, 4 but 4 is not a proper divisor
ex. what are proper divisors of 10 ? _____________________________

common factor: a number that is a factor of two or more numbers is called a common factor of those numbers

Find the factors of 12 \( 12: 1, 2, 3, 4, 6, 12 \)  
Find the factors of 20 \( 20: 1, 2, 4, 5, 10, 20 \)

What is (are) the common factors of 12 and 20 ? _______________________________

GCF: the largest of the common factors of the given numbers is called the greatest common factor.

Find the GCF of 12 and 20 \( \text{GCF} = 12 \)  
Find the GCF of 24 and 36 \( \text{GCF} = 12 \)
Using the prime factorization of the numbers: find the GCF of

Again, factor (prime factorization in exponential form)
– but now use only the factors that are common to both (use the smallest exponent)

20 and 30  \( \rightarrow \) \begin{align*}
20 &= 2^2 \cdot 5 \\
30 &= 2 \cdot 3 \cdot 5
\end{align*}

\( \rightarrow \) \begin{align*}
\text{GCF} &= 2 \cdot 5 = 10
\end{align*}

40 and 48  \( \rightarrow \) \begin{align*}
40 &= 2^3 \cdot 5 \\
48 &= 2^4 \cdot 3
\end{align*}

\( \rightarrow \) \begin{align*}
\text{GCF} &= 2^3 = 8
\end{align*}

Examples 4 and 5 on page 153, 154

ex. 4: Order of diskettes: Customer can only order diskettes in quantities of 20, 50, or 100. How many diskettes should be packages together so that no package needs to be opened when an order is filled?

ex. 5: Running Laps: you run a lap every 3 minutes, your friend runs one lap every 4 min. If you start at the same place and time, when will you both be at the same starting place again.

HW: page 155

2, 11, 20, 28, 30, 39, 48, 56, 57, 58, 61, 62, 63,
Additional examples of GCF and LCD

List the factors (divisors) of
16 ➔ _____________________ 24 ➔ _____________________

Find the largest factor they have in common – this is the greatest common factor ______

List the multiples of
4 ➔ _____________________ 8 ➔ _____________________

Find the least (smallest) multiple they have in common ______
this is the least common multiple

GCG – Greatest common Factor
largest number that will divide a given set of numbers

ex. GCF (12, 8) = 4 GCF (24, 30) = _________

ex. GCF (36, 48) = _________

Note: if the GCF of two numbers is one, we say the numbers are relatively prime ➔ GCF (12, 25) = ___

ex. A carton is to contain 20 small boxes. A second larger carton is to contain 30 of the small boxes
How many boxes should come together “bundled” – so that both cartons can be filled without
having to break each bundle of the smaller boxes?

LCM - Least Common Multiple
least (smallest) number that can be divided evenly by every member of a given set (group) of numbers

ex. LCM (4, 6) = _________ ➔ multiples of 4 and 6: 4, 8, 12, 16, … ➔ 6, 12, 18, 24
ex. LCM (12, 8) = _________
ex. LCM (24, 30) = _________ LCM (36, 48) = ______________
Fractions: describe as a part of the whole

Students: males in class  

budget: part of tuition  

meal: healthy part 

I have run \( \frac{3}{4} \) of a the race  

We spend \( \frac{2}{3} \) of our budget on bills  

The recipe calls for \( \frac{4}{5} \) of a cup of flour

Three parts of a fraction: numerator – denominator, line (vinculum)

\[ \frac{5}{12} \text{ or } \frac{5}{12} \] 

5 is the numerator, 12 is the denominator

Three types of fractions:

proper fraction: a fraction in which the ________________________________

improper fraction: a fraction in which the ________________________________

mixed number: includes a fraction and a whole number part

Note: the fraction bar can be thought of as “division”

ex. We can write whole numbers (and any integer) as fractions: \( \frac{5}{1} \), \( \frac{8}{1} \), ...

ex. What happens when you write a fraction with zero in

a) numerator \( \frac{0}{5} = \) ______  

b) in the denominator \( \frac{5}{0} = \) ______
We can change from an improper fraction to a mixed number

ex. \( \frac{34}{7} = 4 \frac{6}{7} \) since \( 34 \div 7 = 4 \) remainder 6 

ex. \( \frac{43}{5} = 8 \frac{3}{5} \) since \( 43 \div 5 = 8 \) remainder 3

Change to a mixed number:

\[
\frac{45}{7} = \underline{6} \frac{3}{7} \quad \frac{242}{12} = \underline{20} \frac{2}{3} \quad \frac{321}{10} = \underline{32} \frac{1}{10}
\]

Likewise, we can change from a mixed number to an improper fraction

ex. \( 3 \frac{8}{9} \) (you may see it in this form: \( 3 \frac{8}{9} \)) = \( \frac{35}{9} \) (you may see it as \( 3\frac{8}{9} \)) since \( 3 \times 9 + 8 = 35 \)

ex. \( 4 \frac{3}{8} \) (you may see it as \( 4 \frac{3}{8} \)) = \( \frac{35}{8} \) (you may see it as \( \frac{35}{8} \)) since \( 4 \times 8 + 3 = 35 \)

Write in improper form.

\[
\frac{103}{5} = \underline{20} \frac{3}{5} \quad 2 \frac{3}{7} = \underline{\frac{17}{7}} \quad \frac{112}{9} = \underline{12} \frac{4}{9}
\]

We can compare fractions by using the symbols (order relation): >, <, ≤, or ≥

Insert the correct symbol between the fractions;

some may be obvious:

\( \frac{1}{2} \) and \( \frac{7}{9} \) → _______ or \( \frac{4}{8} \) and \( \frac{15}{30} \) → _______

others may require some thought:

\( \frac{3}{11} \) and \( \frac{5}{21} \) ➡ _______ \hspace{1cm} \frac{5}{16} \) and \( \frac{7}{20} \) ➡ _______
Equivalent fractions

Notice that the following shaded regions can be expressed in different form. When two fractions represent the same quantity, we say the fractions are equivalent.

Given a fraction we can change it to an equivalent form and compare it with other fractions

**properties: Given a fraction a/b, b not equal to zero**

1) \( \frac{a}{b} = \frac{ac}{bc} \) \( \Rightarrow \) you can multiply the numerator and the denominator by the same nonzero quantity.

2) \( \frac{a}{b} = \frac{a \div c}{b \div c} \) \( \Rightarrow \) you can divide the numerator and the denominator by the same nonzero quantity.

**Reduce a fraction to simplest form**

1) \( \frac{24}{30} = \) _______  
2) \( \frac{42}{24} = \) _______  
3) \( \frac{-20}{44} = \) _______

**Change it to an equivalent form**

1) \( \frac{3}{5} = \) ? / 35, _______  
2) \( \frac{2}{7} = 10 / ? , \) _______  
3) \( \frac{12}{13} = ? / 39 , \) _______

Example 13 on page 164

Population in US in school:


What fraction of the students in grade 12 and below are in the HS group (grades 9-12)? _______

For every dollar’s worth of product sold by Coca-Cola outside the US, Coke’s profit is 30 cents. What fraction of every dollar’s worth of product sold outside the US is profit for Coca-Cola? ______

HW: Page 165:

2, 6, 7, 13, 20, 28, 31, 42, 46, 50, 57, 66, 72, 78, 81, 88, 90, 98, 100, 111, 119, 122, 126, 127, 133, 135, 140, 142
Addition and Subtraction of Fractions:

Three friends share the driving: Joe drives 2/5 of the way, Bill drives ¼ of the way, and Raul drives the remaining part.

What portion of the drive did Joe and Bill drive? ______________

How much more does Joe drive than Bill? ______________

What portion did Raul drive? ______________

We can only add/subtract fractions if they have a common denominator. We still have the same rules of addition and subtraction as before.

ex. \( \frac{12}{25} + ( - \frac{8}{25} ) = \) _____________

ex. \( -\frac{13}{40} - ( -\frac{8}{40} ) = \) _____________

ex. \( 2\frac{3}{8} + ( + 5\frac{5}{8} ) = \) _____________

If we do not have a common denominator, we write fractions as equivalent fractions with the same denominator. Note: when in doubt – you can find the LCD by taking the product of the given denominators and dividing by the GCF

ex. \( \frac{3}{4} + ( -\frac{2}{3} ) = \) ________

ex. \( ( -\frac{5}{12} ) - ( -\frac{11}{12} ) = \) ________

ex. \( ( 2\frac{1}{2} ) - ( -2\frac{1}{3} ) = \) ________

ex. \( 12 - 8\frac{3}{7} = 12 - \frac{8\frac{3}{7}}{7} = \) ________

Other Examples:

4 3/8 - 2 5/8 = _____________

3 1/6 - 2 1/3 = _____________

HW: page 181:
Products and quotients:

We still have the same rule of signs
1) product (quotient) of two numbers with like signs will be positive
2) product (quotient) of two numbers with different signs will be negative

Note: Sometimes it’s best to change fractions to their improper form.

When we divide – invert the 2nd fraction and multiply by the first – always remember to try to write the fraction in simplest (reduced) form.

Rules –
1. \( a/b \cdot c/d = ac/ bd \), reduce whenever possible
2. \( a/b ÷ c/d = a/b \cdot d/c = ad / bc \), reduce

Examples of fractions

Subtraction & Addition

- \( 4 11/18 - (-2 1/2) = \) ____________  
  \( 5 5/6 + 4 7/8 = \) __________

\[ 4 \frac{1}{2} - 6 1/3 = \] ________________  
\[ 12 \frac{1}{4} - 6 = \] ________

Products

\( 3/8 \cdot 4/5 = \) ________  
\( - a/b \cdot (-b/6) = \) ____________

\( 2/3 \cdot -3/8 \cdot 4/9 = \) ________________  
\( 8 \cdot 1/8 = \) ________________

\( 3 1/2 \cdot 2 \frac{2}{3} = \) ________________  
\( 4 \cdot 2 \frac{1}{3} = \) ________________

Quotients

- \( -5/16 ÷ (3/8) = \) ________________  
  \( ¾ ÷ (-12) = \) ________________

- \( 3 1/3 ÷ 5/8 = \) ________________  
  \( -1 ½ ÷ 1 \frac{1}{4} = \) ________________
One rod is equal to 5 ½ yards. How many feet are in one rod? ____________

A 16 foot board is cut into pieces 2 ½ foot long for use as a bookshelves. What is the length of remaining piece after as many shelves as possible are cut? ________

What is the area of a triangle with the following dimensions? ____________

What is the perimeter of the rectangle with width 12 in. and area 156 in²? ____________

other examples:
1) 4 + 5 3/8 = ____________  2) 6 / 11 + 8/11 = ____________
3) 3 2/3 + 5 2/3 = ____________  4) (- 4 ¼ ) - (- 2 3/8) = ____________
5) 6 • 3/5 = ____________  6) - 4 • 3 / 8 = ____________
7) - 2 ½ • 3 1/3 = ________  8) 4 3/8 ÷ 3 1/2 = ____________
9) 12 / n + (- 2/n) - (+ 3/n) = ____________  10) Is 2/3 a solution of 6x = 4? _______
11) 3/x • 5/y = ____________  12) Evaluate if x = -49 and y = 5/14 xy = ______
13) 8 ÷ 1 1/3 = ____________  14) 3/6 ÷ 0 = ____________
15) 12 ½ ÷ 4 = ________

16. One rod is equal to 5 ½ yard. How many feet are in one rod? ______________

17. A car used 12 ½ gallons of gasoline on a 275 mile trip. How many miles can this car travel on 1 gallon of gasoline.
More Examples of Fractions

1. What is the greatest common factor of
   a) 24 and 32 → GCF (24, 32) = ________  
   b) of 40 and 36 → GCF(40, 36) = ________

2. Find the least common multiple of
   a) 12 and 16 → LCM (12, 16) = ________  
   b) 20 and 15 → LCM(20, 15) = ________

3. Change to an improper fraction
   a) \( \frac{2}{9} = \) ________  
   b) \( -\frac{3}{8} = \) ________

4. Change to a proper fraction
   a) \( \frac{12}{5} + \frac{18}{5} = \) ________  
   b) \( -\frac{5}{3} - \frac{7}{3} - \frac{2}{3} = \) ________

5. Joe ate \( \frac{3}{4} \) of a pie. Bill came and ate \( \frac{1}{4} \) of the remaining pie. What part of the pie is still left? ________

6. You spend \( \frac{3}{5} \) of your total budget on education and \( \frac{3}{4} \) of the amount that you spend on education is for tuition. What fraction of your total budget do you spend on tuition?
7. Addition and subtraction.
   a) $-4 + 2 \frac{3}{5} = \underline{\text{_______}}$
   b) $3 \frac{2}{7} + (-1 \frac{4}{7}) = \underline{\text{_______}}$
   c) $8 + (-3\frac{1}{11}) = \underline{\text{_______}}$
   d) $-2 \frac{3}{4} - (-3 \frac{1}{4}) = \underline{\text{_______}}$
   e) $\frac{a}{5} - 2\frac{a}{5} = \underline{\text{_______}}$
   f) $\frac{7}{c} + (-\frac{2}{c}) = \underline{\text{_______}}$

8. Products and Quotients.
   a) $-4 \cdot 4 \frac{1}{2} = \underline{\text{_______}}$
   b) $-\frac{3}{8} \cdot \frac{4}{15} = \underline{\text{_______}}$
   c) $-2 \frac{2}{3} \cdot 4 \frac{1}{2} \text{ (that's a negative two and two-thirds times four and one-half)} = \underline{\text{_______}}$
   d) $a \cdot \frac{5}{a} = \underline{\text{_______}}$
   e) $\frac{a}{8} \cdot \frac{12}{a} = \underline{\text{_______}}$
   f) $12 \div \frac{3}{4} = \underline{\text{_______}}$
   g) $2 \frac{1}{2} \div 3 \frac{1}{3} = \underline{\text{_______}}
9. Addition and Subtraction without a common denominator – all answers should be in proper form (proper fractions)
   a) \( \frac{3}{4} - \frac{5}{6} = \) ________  
   b) \( \frac{3}{5} + \frac{4}{9} = \) ________
   c) \( 3 - 2 \frac{1}{4} = \) ________  
   d) \( -8 \frac{3}{5} - 4 \frac{1}{2} = \) ________
   e) \( 4 \frac{2}{3} + (-5 \frac{1}{2}) = \) ________

10. A person lost \( 4 \frac{1}{2} \) lbs the first week and then gained \( 2 \frac{1}{2} \) the next week. What was his net gain over the two week period? If he weighed \( 164 \frac{1}{2} \), how much does he weigh now?

11. Find the solution of each of the following equations.
   a) \( \frac{2}{3} + x = \frac{1}{3} \rightarrow x = \) ________  
   \[ \frac{2}{5}x = \frac{1}{4}, \quad \rightarrow \quad x = \] ________

12. Find
   a) \((-2/3)^2 = \) ________  
   \((1 \frac{1}{2})^2 = \) ________
   c) \( \frac{1}{2} - 2/3 \times 3/5 - (1/3) = \) ________
Solving equations with fractions

We still have the same rules

1) add (subtract) equal quantities from both sides of an equation and still have the same solution

2) multiply or divide both sides of the equation by the same nonzero number without changing the solution.

ex. \( x - 4 = 12 \) = ____________  

ex. \( x + 5 = -2 \)

ex. \( 4x = 24 \Rightarrow x = \)  

ex. \( x/4 = 20 \Rightarrow \)

ex. \( x + \frac{3}{4} = \frac{1}{2} \Rightarrow \)  

ex. \( -\frac{2x}{3} = \frac{3}{4} \Rightarrow \)

ex. Two –thirds of a number is equal to four-fifteenths. Find the number

ex. \#33/p.212 \( \Rightarrow \) \( \frac{1}{24} \) of JC Penney’s total sales were from direct marketing services. The sales from direct marketing services were 96 million. Find JC P’s total sales for this period.

ex. \#39/p.212 \( \Rightarrow \)  

The average number of miles per gallon for a truck is calculated by the formulas \( a = \frac{m}{g} \), where \( a \) is the average number of miles per gallon and \( m \) is the total number of miles traveled on \( g \) gallons. Find the number of miles a truck can travel on 38 gal. of diesel fuel if the truck averages 14 miles per gallon.

HW: page 211
Section 3.6: Exponents, complex fractions, and order of operations

We still have the same rules with exponents and order of operations.

\[
\begin{align*}
(\frac{2}{5})^3 &= \underline{\quad}\quad (\frac{-\frac{3}{4}}{3})^3 &= \underline{\quad}\quad -\frac{1}{3}^4 &= \underline{\quad}\quad (2\frac{1}{4})^2 &= \underline{\quad}
\end{align*}
\]

Evaluate \(-x^2y\) if \(x = -2\) and \(y = -3\).

PEMDAS:

- \(\frac{3}{7} \times \frac{14}{15} + \frac{4}{5}\)
- \((-\frac{2}{3})^2 - \frac{7}{18} + \frac{5}{6}\)

\[
\begin{align*}
\text{ex.} \quad \left(\frac{2}{3}\right)^2 + \frac{8}{3} - \frac{7}{8} + \frac{3}{3} - \frac{9}{8} &= \frac{3}{5} - \frac{7}{18} + \frac{5}{6} \\
\text{ex.} \quad \frac{9/10 + 2/5}{1\frac{1}{4}} &= \frac{1\frac{1}{4}}{1} = \underline{\quad} \\
\text{ex.} \quad \frac{2}{3 - 1\frac{1}{2}} &= \underline{\quad}
\end{align*}
\]

\[
\begin{align*}
\text{ex.} \quad \frac{9/16}{3/4} &= \underline{\quad} \\
\text{ex.} \quad \frac{2/3 + 1/2}{7} &= \underline{\quad}
\end{align*}
\]

HW: page 219
Additional work on page 227 (chapter review), page 229 (chapter test), page 231 (cumulative review)
1. Change to a mixed number: \( \frac{38}{7} = \) __________

2. Change to an improper fraction: \( 4 \frac{3}{8} = \) __________

3. Write as an equivalent fraction
   a) \( \frac{3}{8} = \ ? / 40 , \) ________
   b) \( \frac{5}{6} = 20 / ?, \) ________

4. Which is larger? Use the correct symbol; < , > , = .
   a) \( \frac{3}{8} \) .... \( \frac{7}{23} \)
   b) \( \frac{2}{3} \) .... \( \frac{13}{19} \)
   c) \( \frac{1}{5} \) .... \( \frac{8}{40} \)

4. Simplify.
   a) \( \frac{5}{x} + \frac{12}{x} + (- \frac{3}{x}) = \) __________
   b) \( \frac{2}{3} + \frac{1}{2} \)

3. Add – subtract the following
   a) \( (4 \frac{2}{5}) - (6 \frac{7}{10}) = \) ________
   b) \( -\frac{1}{4} + \frac{2}{3} = \) ________
1. The set \{ 0, 1, 2, 3, \ldots \} is called the set of __________________________________________

2. The numbers \{ 2, 3, 5, 7, 11, \ldots \} are called prime numbers because

3. Find the sum of 498 and 1079. ________________

4. Last year 2000 students went to the library without checking out a book. The year before 1849 went and did not check out a book. If this trend continues – same amount of increase in students not checking out a book - then how many students do you expect to go to the library and not check out a book?

5. Find the product of 32 and 27. ____________

6. What is the largest factor of the product 4 \times 12 = 48? ____________

7. A box contains 11 bags of candy. If each bag contains 31 pieces of candy, then how many pieces of candy are in the box?

8. If 4 (7) means 7 + 7 + 7 + 7, then what does 4 (x) mean? ___________________

   What is the value of \( 4^3 \) = _______________
0.1 Homework points - (20, 15, 10) ➔ ______________. The rest is worth 80 points.

1. Write down the set of integers – do not forget the symbol {…. ... }

⇒ ANS. - { … - 2, - 1, 0 , 1, 2, 3, … }

2. Find a prime number less than 30 and greater than 25 ⇒ 29

3. Complete by using the commutative law of addition ( - 2 ) + ( + 4 ) = ⇒ (+4) + (-2) I do not want a single number as the answer.

4. Complete by using the associative law of multiplication: [ (-3) • 4 ] • (- 5) = ⇒ (-3) [(4) (-5)]

5. The sum of any number and its opposite is = ⇒ 0

6. Find

(- 235 ) + ( - 567 ) = ⇒ - 802

(+ 345 ) + ( - 234 ) = ⇒ +111

7. List the positive integers = ⇒ { 1, 2, 3, 4, … }

8. Find each of the absolute values.

a) | 456 | = 456

b) | + 341 | = 341

c) | - 24 | = 24

d) | 0 | = 0

9. Find the opposite of

- 4 ⇒ 4

- ( - 8 ) ⇒ ( - 8) = -8

-c ⇒ c

-43 ⇒ -8

10. Find

(- 24 ) - ( - 35 ) = +11
1. Homework (20, 15, 10, 0) ________________

2. \{0, 1, 2, 3, \ldots\} is called the set of _________________

3. \{\ldots-2, -1, 0, 1, 2, 3, \ldots\} is called the set of _________________

4. Find the solution of each of the following equations.
   a) \(c + 23 = 50\) \(\Rightarrow c = \) ____________  
   b) \(12z = 48\) \(\Rightarrow z = \) ______________

5. The product of ten and a number \(n\) is equal to three hundred. Find the number \(n\). ______________

6. Four thousand represents four hundred fifty plus a number. Find the number. ______________

7. A number increased by fourteen equals forty-eight. Find the number. ______________

8. Four less than a number is 26. Find the number. ______________

9. Simplify to single number.
   a) \(16 - (13 - 5) ÷ 4 = \) ________________  
   b) \(17 + 1 - 8 \cdot 2 ÷ 4 = \) ________________

10. What is the number three units to the right of -4? ______________
1. Which of these two sets is called the set of whole numbers
   \{ 1, 2, 3, 4, \ldots \} or \{ 0, 1, 2, 3, 4, \ldots \} ? ___________________

2. Which of the sets above is also called the set of natural numbers or the set of counting numbers?
   __________________

3. The set \{ 2, 3, 5, 7, 11, \ldots \} is called the set of __________________________.

4. What is the smallest even whole number? __________

5. Use the symbol that best fits the statement <, >, \geq, or \leq
   - 4 _______ - 10  \quad 2010 _______ 2100  \quad 8 _______ 8

6. List the digit that is in
   a) in the hundreds place: 4,568 \rightarrow ________  b) in the millions place: 56,345,123 \rightarrow ________

7. Write down the number in standard form
   a) four hundred three \rightarrow __________  b) one thousand three hundred forty \rightarrow __________

8. Write the following number in words: 34,501 \rightarrow 
   ___________________________________________________

9. Round: 8053 to the nearest hundred \rightarrow __________

    Round: 19,512 to the nearest thousand \rightarrow __________
0.1. HW credit ________________

1. Find the smallest natural/counting number. ________________

2. Find the smallest prime number. ________________

3. Round to the nearest thousand, 45,499 \(\Rightarrow\) ________________

4. Find the sum of 456 + 861 = ________________
   What is the result of 498 + 578 = ________________
   What is the new number when 43 is added to 135? ________________

5. Find the value of c when 467 + c = 500, c = ____________

6. Estimate the sum of 582 and 409 \(\Rightarrow\) ________________

7. Find c + 0 = ____________ (property of zero)
   a + b = ____________ commutative law of addition
   (a + b) + c = ____________ associative law of addition

8. Is 2 a solution of the equation 6 + x = 4? ________________

9. The circle graph represents the total number of students from each area.

   70 From West Texas  15 From Central Texas
   5 From East Texas  10 from outside of Texas

   What is the total number of students? ______
   What is the total number of students from Texas? ____