Chapter 5
Algebraic Fractions – fractions of polynomials.
You will need to recall how to factor polynomials when you are working with fractions.

Multiply:
1) \((x - 2y)(x + 2y) = \) _________________
2) \((2x + 3y)(4x^2 - 6xy + 9y^2) = \) _________________
3) \((2x - 3y)(x + 2y) = \) _________________

Factor each of the following:
4) \(x^2 - 25 = \) ________________
5) \(x^2 + 16 = \) ________________
6) \(8x^3 - 2x^3 = \) ________________
7) \(x^4 - 8x^2 + 16 = \) ________________
8) \(2x - xy + 4y - 8 = \) ________________

Factor each of the following trinomials:
\(x^2 + 8x + 7 = \) ________________
\(x^2 - 5x + 4 = \) ________________
\(x^2 - 8x + 12 = \) ________________
\(x^2 + 10x + 21 = \) ________________
\(x^2 + 4x - 15 = \) ________________
\(x^2 - 6x - 16 = \) ________________
\(x^4 - 2x^2 - 8 = \) ________________

Case II – Factor the trinomials when \(a \neq 1\)
\(2x^2 - 3x + 1 = \) ________________
\(3x^2 - 5x + 2 = \) ________________

\(4x^2 - 12x + 5 = \) ________________
\(4x^2 - 5x - 21 = \) ________________
More Examples

Multiply:
1) \((x - 3y)(x + 3y) = \) 

2) \((2x + 5y)(4x^2 - 10xy + 25y^2) = \) 

3) \((2x - 5y)(x + 5y) = \) 

Factor each of the following:
4) \(x^2 - 49 = \) 

5) \(x^2 + 49 = \) 

6) \(12y - 3y^3 = \) 

7) \(2x^4 - 2xy^3 = \) 

8) \(x^4 + 6x^2 + 9 = \) 

9) \(2x - xy + 6y - 12 = \) 

Factor each of the following trinomials:
\(x^2 + 9x + 8 = \) 
\(x^2 - 6x - 10 = \) 
\(x^4 - 2x^2 - 3 = \)
Fundamental Principle of Fractions
\[
\frac{a}{b} = \frac{ax}{bx}, x \neq 0
\]

This means we can multiply a fraction by any nonzero number without changing the value.

We can also look at it like
\[
\frac{ax}{bx} = \frac{a}{b}, x \neq 0
\]

This is what we use to reduce a fraction.

To simplify (reduce) a fraction – means to write the fraction in a form in which the numerator and the denominator have no factor in common.

Reduce \( \frac{20}{35} \). We can write \( \frac{20}{35} = \frac{4 \cdot 5}{7 \cdot 5} = \frac{4}{7} \). This may not be the way you usually do it – but this is the reason behind it.

Note: You can cancel factors but you can not cancel terms.

example: \( \frac{2x}{3x} \), we can cancel the x’s because they are factors (multiplied)

\[
\frac{2 + x}{3x}, \text{ you can not cancel the x’s because x is a factor in the denominator but it is a term in the numerator.}
\]

Simplify. Factor and then cancel as much as possible.

1. \( \frac{3x}{x^2 - 2x} = \)  

2. \( \frac{4x + 2}{12x + 6} = \)

3. \( \frac{x + 2}{x^2 - 4} = \)

4. \( \frac{x^2 - 3x}{x^2 - 4x + 3} = \)
5. \[ \frac{9x^2 - 16}{3x^2 - 16x + 16} = \] 

6. \[ \frac{6x^2 - x - 2}{6x^2 + 5x - 6} = \]

Since we are talking about fraction – operations that we have done before are acceptable here. We can multiply, divide, add, and subtract fractions.

**Multiplication and Division**

Recall that

\[
\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad \text{(reduce when possible)}
\]

Also,

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \quad \text{(reduce when possible)}
\]

Same idea with algebraic fractions.

**Process:**
1. Factor all numerators and denominators into prime factors (factor)
2. Cancel any factor that is common to a numerator and a denominator
3. Multiply the remaining factors in the numerator and place this result over the product of the remaining factors in the denominator.

**Examples:**

1. \(\frac{1}{203}\)

\[ \frac{2x + 2}{x + 3} \cdot \frac{x + 3}{2x - 6} = \]

2. \(\frac{3}{203}\)

\[ \frac{x^2 - 1}{5x - 5} \cdot \frac{5}{x^2 + 5x + 4} = \]

3. \(6x \div \frac{2x}{x + 2}\)
4. \( \frac{x^2 + 8x + 15}{x^2 - 4x - 21} \div \frac{x + 5}{x^2 - 3x - 28} = \) 

5. \( \frac{4x^2 + 12x + 9}{3x^2 + 2x - 1} \div \frac{4x^2 - 9}{3x^2 + 14x - 5} = \) 

6. \( \frac{25x^2 - 1}{10x^2 + 17x + 3} \div (1 - 5x) = \)
Remember that when we add, you can combine terms that are similar. When we work with fractions, similar idea works – you must have a common denominator (same denominator) before you can add or subtract.

\[ 2x + 5x = 7x \]
\[ \frac{2}{x} + \frac{5}{x} = \frac{2+5}{x} = \frac{7}{x} \]

\[ 12ay - 24ay = -12ay \]
\[ \frac{2x}{y} - \frac{4x}{y} = \frac{2x - 4x}{y} = \frac{-2x}{y} \]

Simplify by adding and subtracting – reduce whenever possible.

1. \[ \frac{3}{8} + \frac{1}{8} = \frac{4}{8} \]
2. \[ \frac{2}{x} + \frac{y}{x} = \frac{2+y}{x} \]

3. \[ \frac{2}{y} + \frac{a}{y} - \frac{3}{y} = \frac{2+a-3}{y} = \frac{a-1}{y} \]
4. \[ \frac{2}{a} - \frac{a-2}{a} = \frac{2-a+2}{a} = \frac{4-a}{a} \]

5. \[ \frac{x}{x^2-1} + \frac{1}{x^2-1} = \frac{x+1}{x^2-1} \]
6. \[ \frac{3x}{x+3} - \frac{x-6}{x+3} = \frac{3x-(x-6)}{x+3} = \frac{2x+6}{x+3} \]

What if the fractions do not have a common denominator? 

\[ \frac{3}{5} + \frac{1}{4} = \]
\[ \frac{2}{x} - \frac{3}{2} = \]
You need to have a least common denominator (LCD). We worked with LCM – that’s what this is.

Find the LCM of

16, 24x → _______ 2x, 3y → __________

x, x + 2 → _______ x + 4, x + 2 → __________

x + 3, x² – 9 → _______ x² – 9, x² + 4x + 3 → __________

Examples:

Find the least common denominator of

\[ \frac{1}{a-2} \text{ and } \frac{2a}{a^2 - 3a - 10} \] \[ \frac{1}{x} \text{ and } \frac{4}{x + 2} \]

\[ \frac{x - 1}{x^2 + 5x + 6} \text{ and } \frac{x}{x^2 - x - 12} \]
Find the missing part.

1. \( \frac{2x + 3}{x - 4} = \frac{?}{x - 4} \)

2. \( \frac{2x + 1}{x^2 - 3x - 28} = \frac{?}{(x - 7)(x + 3)(x + 4)} \)

3. \( \frac{x + 2}{x - 1} = \frac{?}{x^3 - 1} \)

Now that we know how to find the LCD and how to change a fraction to an equivalent form – let’s look at some examples of addition and subtraction.

Simplify by performing the given operation.

1. \( \frac{2}{215} \)

\( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \) \___________

2. \( \frac{4}{215} \)

\( \frac{2}{x} + \frac{1}{x + 1} = \) \___________

3. \( \frac{7}{215} \)

\( \frac{2}{x + 2} + \frac{1}{x - 5} = \) \___________

4. \( \frac{3}{x - 2} + \frac{5 - x}{2 - x} = \) \___________
When you subtract – be careful with the signs on the fraction that follows the subtraction.

5. \( \frac{3}{219} \) 
   \[ \frac{x}{x+1} - \frac{2}{3} = \]

6. \( \frac{6}{219} \) 
   \[ \frac{1}{x+2} - \frac{1}{x+3} = \]

7. \( \frac{12}{220} \) 
   \[ \frac{x+2}{x-5} - \frac{5x+31}{x^2-2x-15} = \]

8. \( \frac{14}{216} \) 
   \[ \frac{x+3}{x-6} + \frac{x-1}{x^2-2x-24} = \]
We still have order of operations – if there is a mixture of operations, follow the order of operations agreement.

1. $\frac{3}{x^2} + \frac{5}{y} \cdot \frac{4y}{3x} = \underline{\hspace{2cm}}$

2. $\frac{2}{x+5} + \frac{1}{x-4} - \frac{3}{x+3} = \underline{\hspace{2cm}}$

3. \[
\frac{1}{x^2-x-2} \div \left( \frac{1}{x+1} + \frac{3}{x-2} \right)
\]

4. \[
\frac{1}{x^2-x-6} \div \left( \frac{1}{x+2} - \frac{1}{x-3} \right)
\]
Sometimes we find fractions within a fraction – complex fraction.

Methods:
1. Find LCD of all fractions in the expressions. Multiply the numerator and the denominator of the complex fraction by the LCD. Simplify.
or
2. Use order of operation by simplifying each group of fractions in the proper order.

Examples:
1. 1/224
\[
\frac{\frac{1}{a}}{\frac{1}{a^2}} = \frac{1/a}{1/a^2} = \text{__________}
\]

2.
\[
\frac{4/x}{6} = \text{_____}
\]

3.
\[
\frac{1}{x} - \frac{1}{y} = \text{__________}
\]

4. 10/225
\[
\frac{\frac{1}{3} + \frac{1}{a}}{\frac{1}{b}} = \text{__________}
\]

6. 12/225
\[
\frac{1}{x + 2} - \frac{1}{x} = \text{_______}
\]

7. 19/226
\[
\frac{\frac{1}{x + y} - \frac{1}{y}}{\frac{1}{x^2 - y^2}} = \text{__________}
\]
Now that we have worked with fractions let’s look at equations with fractions. We combine the idea of working with fractions and the idea of solving equations.

**Steps.**
1. Find LCD of all fractions.
2. Multiply every term of the equation by LCD
3. Combine similar terms on each side of the equation
4. Use the previous rules of solving equations with respect to adding, subtracting, multiplying, and dividing.

**Examples**

1. \( \frac{3}{2x} = \frac{4}{5} \)
2. \( \frac{1}{x} - \frac{3}{2x} = 5 \)

3. \( \frac{6}{232} \quad \frac{4}{x} + \frac{1}{2x} = \frac{9}{4} \)

4. \( \frac{1}{x+1} = \frac{2}{1-x^2} \)

5. \( \frac{14}{233} \quad \frac{1}{x^{2}-3x} = \frac{2}{x^{2}-9} \)

6. \( \frac{17}{233} \quad \frac{x}{x+4} = \frac{x}{x-4} = \frac{x+18}{x^{2}-16} \)
Word Problems with fractional equations (equations with fractions)

1. 22/233  
   In a fraction the numerator is three less than the denominator. If one is added to both the numerator and the denominator, the value of the resulting fraction is 5/6. Find the original fraction.

2. 27/234  
   An experienced bricklayer and his apprentice can build a wall together in 3 hours. It would take the apprentice 12 hours to do the job alone. How long would it take the experienced bricklayer to do the job alone?

3. 30/235  
   A drain can empty a tank in 4 hours. Pump A can fill the tank in 6 hours and pump B can fill it in 8 hours. If both pumps are working together and the drain is accidentally left open, how long will it take to fill the tank?
Chapter Test and Review on page 236 – 245.
Chapter Six
More on Exponents and Radicals.

We have worked with a couple of the exponent laws—let’s complete the laws and use in general form. In order to simplify the division rule—let’s work with the idea of negative exponents first.

Def. For any real number \( x \), \( x \neq 0 \), \( x^0 = 1 \)

Recall that

\[
4^0 = 1, \quad (-4)^0 = \_
\]

\( -4^0 = -(4)^0 = \_
\)

and \( 0^0 = \_
\)

Def. Let \( n \) be a natural number. We define \( x^{-n} = \frac{1}{x^n} \).

We can talk about expressions with negative integer exponents—because we a way to change the expression to one with positive exponents.

\[
3^{-2} = \frac{1}{3^2} = \_
\]

\( -4^2 = -(4)^2 = \_
\)

\[
4^{-3} = \_
\quad \frac{4}{3^{-2}} = \_
\]

Complete set of Rules of Exponents for integers. They worked with natural numbers but they also work with integers.

If \( x \) and \( y \) represent any real number \( \neq 0 \) and \( n \) and \( m \) are any integers,

1. then \( x^n \cdot x^m = \_
\)

2. then \( \frac{x^n}{x^m} = x^{n-m} \)

3. \((x^n)^m = \_
\)

4. \((xy)^n = \_
\)

5. \(
\left(\frac{x}{y}\right)^n = \_
\)
Examples of Rules with negative, positive, and mixed exponents.

1. Rewrite using only positive exponents and simplify.
   
a) \((2x)^2 = \) ________
b) \(4^{-2} = \) ________

c) \((2 + b)^{-1} = \) ____________
d) \(a^{-1} + b^{-1} = \) ____________

e) \(2x^{-4} \cdot x^{10} = \) ____________
f) \((3x^{-2})^{-3} = \) ____________

g) \(\frac{x^{-4}}{x^9} = \) ____________
h) \(\frac{4^2 \cdot 4^0}{4^5} = \) ____________

2. More examples with exponents.
   
a) \(\frac{x^{-4} y^3}{2x^{-6} y^4} = \) ____________
b) \(\left(\frac{2 \cdot 4^6}{7^2}\right)^0 = \) ____________

c) \((-2x^{-3})^{-2}(3x^{-1})^2 = \) ____________
d) \((x^{-1} + 2^{-3})^{-1} = \) ____________

e) \(\frac{x^{-1} + y^{-1}}{1^{-1}} = \) _____
One common use of exponents in science in when we use numbers in what is called scientific notation.

A number is in scientific notation if it is expressed as the product of a power of 10 and a number equal to or greater than 1 and less than 10.

\[ r.ddddd \times 10^n \]

1. State whether the following number is in scientific notation or not.

0.05 \times 10^8 
7.8 \times 10^{-2} 
5 \times 10^8 
0.645 \times 10^{-3}

2. Write in scientific notation.

0.000034 = \_______________ 
23400 = \_______________

3. Write each of the following numbers without using exponents.

2.34 \times 10^4 = \_______________ 
1.2 \times 10^3 = \_______________

9.8 \times 10^1 = \_______________

4. \frac{37}{262}

5. \frac{39}{262}
Radicals

An expression of the form $\sqrt[n]{x}$ is called a radical expression.

$\sqrt[n]{x}$ is a radical sign, $n$ is called the index (radical index),
and $x$ (quantity inside the radical) is called the radicand.

Suppose we want to find a number so that when squared you get 16 → What is that number? _________

a number so that when squared gives you 81 → What is the number? ______________

Is there more than one possibility for either of these questions? ____________

Because of this we use the radical symbol to express a single value to the two questions above as well to
the general question about how many roots a number has.

We say $c$ is a square root of $x$ if $c^2 = x$.

To ask the question and get a single answer (when such exists – sometimes they do not exist)
we use the following notation

$\sqrt{x} = c$ and read it as “the square root of $x$ is equal to $c$”,
again this means “what number when squared
when there are two possibilities we use the positive root – and call it the principal root.

Anytime you see the radical ($\sqrt{\phantom{x}}$ or $\sqrt[3]{\phantom{x}}$) that means you want the principal square root.

Find each of the following square roots

$\sqrt{121} = $ __________  $\sqrt[3]{1600} = $ __________  $\sqrt{-9} = $ __________
$\sqrt{0} = $ __________  $\sqrt[3]{1} = $ __________

Similar idea goes with cube roots, 4th roots, and in general nth roots.

Cube roots

$\sqrt[3]{x} = c$ if $c^3 = x$ we say $c$ is a cube root of $x$ (we mean the principal cube root – but there is only one)

$\sqrt[3]{27} = $ __________  $\sqrt[3]{-8} = $ __________  $\sqrt[3]{8} = $ __________

$\sqrt[3]{125} = $ __________
Anything after cube roots, we normally say the \( n \)th root \((4\text{th root, } 5\text{th root}...)\)

**fourth roots \((4\text{th roots})\)**

\[
4\sqrt{x} = c \quad \text{if} \quad c^4 = x \quad \text{we say \( c \) is a 4\text{th root of} \( x \)}
\]

(we mean the principal 4\text{th root} – here there are two possibilities so must say the principal 4\text{th root}.)

\[
\frac{4}{4}\sqrt{16} = \_\_\_\_\_
\]

\[
\frac{4}{4}\sqrt{81} = \_\_\_\_
\]

\[
\frac{4}{4}\sqrt{1} = \_\_\_\_
\]

\[
\frac{4}{4}\sqrt{-81} = \_\_\_\_
\]

**Other Examples**

\[
\frac{3}{3}\sqrt{1} = \_\_\_\_
\]

\[
\frac{3}{3}\sqrt{-128} = \_\_\_\_
\]

\[
\frac{3}{3}\sqrt{7^{12}} = \_\_\_\_
\]

Most roots are not “nice” – whole numbers, exact. For example \(\sqrt{3} = ?\)

We either have to use a calculator to get an approximation or leave the radical alone and say “that’s the exact value”.

We have seen literal expressions with integer exponents. Can we have an exponent in fractional form and if so what does it represent?

**Def. Let be a natural number. Then \( x^{\frac{1}{n}} = \sqrt[\frac{1}{n}]{x} \).**

This gives an alternate way to write a radical.

So,

\[
x^{\frac{1}{2}} = \sqrt[\frac{1}{2}]{x} = \sqrt{x}
\]

This makes sense in how we defined radicals and the use of exponent rules.

\[
\sqrt{x} = c, \quad \text{means} \quad c^2 = x, \quad \text{if} \quad x^{\frac{1}{2}} = \sqrt{x}, \quad \text{that would mean that} \quad (x^{\frac{1}{2}})^2 = x. \quad \text{This is true.}
\]
examples:

1. Change to radical form

\[ 4^{\frac{1}{4}} = \boxed{\quad} \quad x^{-\frac{1}{4}} = \boxed{\quad} \quad (-3)^{-\frac{1}{2}} = \boxed{\quad} \quad -4^{\frac{1}{5}} = \boxed{\quad} \]

2. Change from radical to exponential form.

\[ \sqrt{x} = \boxed{\quad} \quad \frac{1}{\sqrt{2}} = \boxed{\quad} \quad \sqrt[3]{5} = \boxed{\quad} \]

Do exponent have a meaning \( x^{\frac{2}{3}} \)?

Def. We define \( x^{\frac{n}{m}} = \sqrt[m]{x^n} \) or \( x^{\frac{n}{m}} = \left(\sqrt[n]{x}\right)^m \) for us these two are equal. We use whichever form appears to be the easiest.

Other Examples:

1. Change to radical form

\[ a^{\frac{4}{5}} = \boxed{\quad} \quad 9^{-\frac{2}{3}} = \boxed{\quad} \quad (2x)^{\frac{2}{7}} = \boxed{\quad} \quad (4^4)^{\frac{1}{6}} = \boxed{\quad} \]

2. Change to exponential form

\[ \sqrt[3]{4^2} = \boxed{\quad} \quad \left(\frac{3}{\sqrt{9}}\right)^2 = \boxed{\quad} \quad \frac{1}{\sqrt[5]{x^2}} = \boxed{\quad} \]
3. Other problems - find the value of (evaluate)

\[ 9^{1/2} = \underline{\phantom{0}} \quad (\text{-}27)^{4/3} = \underline{\phantom{0}} \quad -4^{3/2} = \underline{\phantom{0}} \]

\[ -\left(16\right)^{3/4} = \underline{\phantom{0}} \quad \left(\frac{1}{16}\right)^{-3/4} = \underline{\phantom{0}} \quad \frac{1}{\left(-25\right)^{1/2}} = \underline{\phantom{0}} \]

\[ \sqrt[5]{64} = \underline{\phantom{0}} \quad \sqrt{239^2} = \underline{\phantom{0}} \]

Property of radicals:

1. \[ \sqrt[\text{n}]{ab} = \sqrt[\text{n}]{a} \cdot \sqrt[\text{n}]{b} \]

We know that \[ \sqrt{36} = 6, \]

Notice that \[ \sqrt{36} = \sqrt{4 \cdot 9} \]. According to this property \[ \sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6 \]

This is not how we use the property – but it shows you that it does work.

2. For similar reasons we have,

\[ \sqrt[\text{n}]{\frac{a}{b}} = \frac{\sqrt[\text{n}]{a}}{\sqrt[\text{n}]{b}} \]

both of these properties can be used when the radicand is not a perfect root (perfect square, cube, ...)

91
4. Simplification of radicals

A radical is in simplest form when no power of a factor of the radicand is equal to or greater than the index number.

Process: write radicand in prime factorization form – then all exponents inside the radical must be smaller than the index.

Examples:

\[ \sqrt{20} = \text{______} \quad 5\sqrt{18} = \text{__________} \]

\[ \sqrt[3]{40} = \text{______} \quad \sqrt[3]{9} = \text{__________} \]

\[ \sqrt{x^7} = \text{______} \quad \frac{3}{5} \sqrt{x^9 y^{12}} = \text{__________} \]

\[ \sqrt{18x^7} = \text{______} \quad \frac{3}{5} \sqrt{8x^5 y^{15}} = \text{__________} \]

5. Operations with radicals ( +, −, x, ÷ )

We still have a order of operations as before and --- some limitations on additions and products.

a) you can only add terms when you have the same radicand and the same index (similar for subtraction)

b) you can only multiply when you have the same index ( similar for division)

a) \[ \sqrt{2} + \frac{3}{2} \sqrt{2} = \text{__________} \] 

b) \[ \sqrt{2} + \sqrt{3} = \text{__________} \]

c) \[ 2 + \sqrt{8} = \text{__________} \]

d) \[ 4\sqrt{5} - 7\sqrt{5} \]

e) \[ \sqrt{8} - 2\sqrt{54} + 5\sqrt{3} = \text{__________} \]

f) \[ 4(1 - 3\sqrt{3}) - 2(3 + \frac{3}{2}) = \text{__________} \]
More Complex Radical Expression (fractions) (products and quotients) – including product of “binomials” – Foil method, perfect squares

a) $\sqrt{2}\sqrt{7} = \underline{\phantom{0000}}$

b) $(3\sqrt[1]{4})(4\sqrt[1]{4}) = \underline{\phantom{0000}}$

c) $(4\sqrt[12]{a})(3\sqrt[18]{a^3}) = \underline{\phantom{0000}}$

d) $(\sqrt[2]{54a^4})(2\sqrt[16]{-16a}) = \underline{\phantom{0000}}$

e) $4\sqrt[3]{3}(3\sqrt[6]{6} + \sqrt[15]{15}) = \underline{\phantom{0000}}$

f) $\sqrt[2]{2}(2\sqrt[4]{4} - 2\sqrt[32]{32}) = \underline{\phantom{0000}}$

g) $(\sqrt[2]{2} + \sqrt[5]{5})(\sqrt[3]{3} + \sqrt[2]{2}) = \underline{\phantom{0000}}$

h) $(2\sqrt[5]{5})^2 = \underline{\phantom{0000}}$

i) $(2\sqrt[3]{5} - 4)^2 = \underline{\phantom{0000}}$

j) $(3 - \sqrt[7]{7})(3 + \sqrt[7]{7}) = \underline{\phantom{0000}}$
Radicals in Denominator
When we say a radical expression is in simplest we add one more condition – no radical in the denominator. (Rationalizing the denominator → removing radicals from the denominator)

One term in the denominator

a) \( \frac{2}{\sqrt{6}} = \) ____________

b) \( \frac{3}{\sqrt{20}} = \) ____________

e) \( \frac{2}{\sqrt[3]{4}} = \) ____________

d) \( \frac{4}{\sqrt[3]{9x^3}} = \) ____________

e) \( \frac{3xy}{\sqrt[6]{8x^3y^2}} = \) ____________

f) \( \frac{\sqrt{5}}{\sqrt[7]{7}} = \) ____________

g) \( \frac{8}{\sqrt{x + 2}} = \) ____________

Two Terms in the denominator

a) \( \frac{2}{\sqrt{2} + 2} = \) ______________

b) \( \frac{4}{2 - \sqrt{3}} = \) ______________

c) \( \frac{\sqrt{2}}{\sqrt{2} - 3} = \) __________

d) \( \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \) ____________
1. Simplify. Write without negative exponents.

\[-2^{-4} = \quad \quad 0^{-4} = \quad \quad -4^0 = \quad \quad \]

\[(2^{-1} - x^{-1})^{-1} = \quad \quad \]

2. Write in scientific notation.

\[34.1 = \quad \quad 0.0201 = \quad \quad \]

3. Find

\[\sqrt[4]{400} = \quad \quad \sqrt[4]{4 \cdot 49} = \quad \quad \sqrt[4]{-4} = \quad \quad \]


\[\sqrt{\frac{9}{25}} = \quad \quad \sqrt[3]{-8} = \quad \quad \sqrt[4]{625} = \quad \quad \]

5. Write in simplest form.

\[\sqrt{x^8 y^4} = \quad \quad \sqrt[3]{8x^9 y^{15}} = \quad \quad \]

\[\sqrt[6]{x^2} = \quad \quad \sqrt{20} = \quad \quad \]

\[4 + 3 \sqrt{9} = \quad \quad \sqrt[3]{x^{14}} = \quad \quad \]