Review of relations and functions

Sets:
A set is a collection of numbers or objects that have a well-defined property in common.

Ex. Set of all smart students in class - ____________
Ex. Set of all students with long hair - ____________
Ex. Set of all good fruit - _______________
Ex. The set of all students enrolled in math 1312.050 on Jan. 25 during the spring semester of 2005.

We can list all members of this set whenever possible: { ________ }

We can use what’s is called set-builder notation.
{ x | x represents a student enrolled in math 1312.050 – spring 2005 on Jan. 25}

In some cases – one form may be better than the other.
{ a, e, i, o, u }     { x | x represents a student currently enrolled at A.S.U }

The members of the set are called elements of the set. We use the symbol ∈ to express the fact that an object is a member of a set.

capital letters: sets                                   lower case letters: elements
A = { a, b, c }     or    B = { 5, 10, 15 ..... }

We say   a ∈ A,   5 ∈ B,   d ∉ A, or    7 ∉ B

We will discuss sets in more detail at a different time.

Sets of numbers:

N = set of natural numbers = set of counting numbers = set of positive integers
   = \{ x | x ∈ set of natural numbers \} = \{ 1, 2, 3, 4, .... \}

W = set of whole numbers = set of nonnegative integers
   = \{ x | x ∈ set of whole number \} = \{ 0, 1, 2, 3, ... \}

I = set of integers =
   = \{ x | x ∈ set of integers \} = \{ ... -3, -2, -1, 0, 1, 2, 3, ... \}

Q = set of rational numbers
   = \{ x | x ∈ set of rational numbers \} = \{ m/n | m and n are integers with n ≠ 0 \}

We can not list the members of this set as nicely as we did the first three. The best we can do is
say the above statements and provide examples.
- 3 = -3/1,  0 = 0/6,  4 = 4/1,  2/3, -5/17, 0.24, 0.11111... (1/9), ...

Q’ = set of irrational numbers \rightarrow examples:
π, e, \sqrt{2}, 0.1010010001..., ...

R = set of real numbers : consists of the set of rational and irrational number – no other number
We want to remember the idea of a rectangular coordinate system. I will assume previous knowledge and remind you of the basics.

The plane (flat surface) is separated into four parts; Quadrants I, II, III, IV. Label these quadrants.

Where is the origin at? Every point on the plane can be labeled in the form \( P(x,y) \).

Plot the following points:

\[ A(2, -4), \quad B(-5, -7), \quad (0, -8), \quad \text{and} \quad (4, 0) \]

You have learned how to graph

\[ 2x - y = 4 \quad \quad \quad \quad y = -5 \quad \quad \quad \quad y = x^2 + 2x - 3 \]

In case you have forgotten, I will give you a quick reminder (later).

For now:

Whenever possible – try to use the \( x \) and \( y \)-intercepts - slopes, vertices – if such things exist

1) \( 2x - y = 4 \)
   - \( x \)-intercept: \( x = 2 \)
   - \( y \)-intercept: \( y = -4 \)
   - slope: \( m = 2 \)

2) \( y = -5 \):
   - this is a horizontal line → crosses the \( y \)-axis at \( y = -5 \), \( x \)-intercept = none, slope \( m = 0 \)

3) \( y = x^2 + 2x - 3 \) →
   - represents a parabola, with \( y \)-intercept = 3, \( x \)-intercept = ? ___ vertex = ? _______
We will use some of the basic equations and their corresponding graphs to illustrate the ideas that follow.

**Relations:**

*Def.* A relation is a correspondence between two sets (expresses a relationship between two groups of numbers or objects).

We can use (1) sets to describe them, (2) graphs, or (3) equations.

**Functions:**

*Def.* A special kind of relation: A relation is called a function provided every $x$ produces only one $y$.

That is, once we select a value of $x$ (value of first set, of the domain), there can only be one $y$ (value of the second set; range) that corresponds to the chosen $x$.

Each relation and/or function has a domain and a range.

- **Domain:** Set of all permissible values of $x$.
- **Range:** Set of all values of $y$ that result from using all permissible values of $x$.

**ex.** A group of people that have been married at least once (to each other) is provided. List the members in a relation form so that the first set consists of all women and the second set consists of all men. Use marriage as the connection between the two sets.

Based on your drawing; is it a function or a relation?

**ex.** Suppose that you have five workers that list the number of hours worked in the first set and the amount earned in the second set. Write a relation that represents this situation. Is this a function or just a relation?

**ex.** You buy stamps based on how much it will cost to mail an item. Write some kind of relation that represents the connection between the weight of the object and its cost to mail.

$0.37$ for the first oz, an additional $0.23$ oz. thereafter.
ex. A polluted creek is to be cleaned. Write a relation that expressed the cost of cleaning as it relates to how “clean – pure”.

ex. An item has a particular cost – Find an expression that represents the relationship between the number of items produced and the cost associated with producing that many items.

Now that we have an idea of relations and why we need them – let’s back up and look at them some more from a set, graph, or equation point of view.

1) Sets

A

2

3

-2 → 1

-4

B

1

4

1

-1

-2

C

1

π

√2 → 2

→ __________________       → __________________       → __________________

a) Functions or relations:   A: _______  B:  _____       C = ________

b) Domain:   A: ________________   C : ________________________

c) range:   A :  ________________   B : _____________________

2) Graphs

A

\[ 2x - y + 3 = 0 \]

B

\[ y^2 = 4x \]

C

\[ x^2 + y^2 = 9 \]

a) Functions or relations:

A: __________________  B: __________________  C: _______________

b) Domain:

A: _______________

B: _______________

C: _______________

c) Range:

A: _______________

B: _______________

C: _______________

(3) Other examples in equation form

( other than lines, parabolas, exponential, logarithms, and absolute values)

a) \( f(x) = \frac{2x}{x-2} \)

b) \( y = |2x - 4| \)

c) \( x = 4 \)

d) \( f(x) = \begin{cases} 2x - 1 & \text{if } x > 4 \\ x^2 & \text{if } x \leq 4 \end{cases} \)

Function: __________________________  Relations: __________________

Domain:

A: _______________

B: _______________

C: _______________

D: _______________
Regardless of how we see a relation – always remember the idea of a function.

Which of the above represent the idea of a relation ? __________  Which one represent the idea of a function ? _____

Two important ideas that we will consider along the way.

slopes and areas.

ex.  Find the slope of

\[3x - 4y = 5\]  \[x = 5\]  \[y = x^2 - 2x + 5\]

ex.  Find the area shaded below

\[f(x) = 4\]  \[y = 3x + 6\]  \[P(x) = 4x^4 + 3x^3 - 3x + 7\] (graph may be similar)

\[f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\]
Other Special Functions:

Lines (linear functions)

Remember the general equation $ax + by = c$ What type of curve does this represent? ____________________

Examples of the above type of curves
( slant, vertical, horizontal )

$x + 5y = 2 \implies $ ____________________  
$2x - y = 3 \implies $ ____________________

$2x + 0y = 6 \implies $ ____________________  
$0x - 4y = 12 \implies $ ________________

Any equation of the form $Ax + By = C$ where $B \neq 0$ can be written in the form $y = mx + b$,
In this form, $m$ is called the slope, and $b$ is the y-intercept of the line. Lines may or may not have $x,y$-intercepts.

ex. $y = -3x + 4 \rightarrow$ slope: _________  y-intercept: ___________  what is the x-intercept? _______

ex. $2x + 3y = 5 \rightarrow$

Find the $x$ and the $y$ intercepts of

a. $y = 2x - 4$  
$x$-int = ________  $y$-int = __________

slope: __________

b. $x = 4$  
$x$ int = __________  $y$-int = __________

slope: __________

ex. Let $f(x) = 2x - 5$ , What is the graph of this equation? _________________

find $f(-2) =$ _____ , $f(\frac{1}{4}) =$ ________________

*** $f(h) =$ ________________  ***$f(h + 2) =$ ________________

For what value of $x$ is $f(x) = 0$ ? ___________
Conclusion:

1. \( ax + by = c \) is called a linear equation. Depending on the values of \( a \), \( b \), and \( c \), the line is either

   - **vertical with an undefined slope**: \( x = -2 \), or \( x = 2/3 \), or ...
   - **horizontal with slope zero**: \( y = 5 \), or \( y = \sqrt{2} \), or ...
   - **slant with slope** \( m = -\frac{a}{b} \): can be obtained by writing equation in the slope-intercept form

     \[
     y = 2x - 3, \quad \text{or} \quad y = -\frac{1}{2}x + 7, \quad 2x - 5y = 3
     \]

2. The slope can be found by using \( m = \frac{y_2 - y_1}{x_2 - x_1} \) or if given an equation, rewrite in the slope-intercept form; \( y = mx + b \)

   - ex. slope of the line passing through the points \( (3, -2) \) and \( (4, 3) \) \( \rightarrow \) _______

3. **parallel lines have equal slopes**

   \( y = 3x - 7 \) and \( 2x - 14y = 1 \) are parallel lines. Why?

   Are \( 2x - y = 4 \) and \( x - 2y = 1 \) parallel?

   Note: If two lines are parallel, then there is a relationship between the coefficients of one equation and the coefficients of the second equation.

4. **perpendicular lines have negative reciprocal slopes**

   \( 2x - y = 3 \) and \( y = \frac{1}{2}x + 2 \) are perpendicular. Why?

   What is the slope of a line that is perpendicular to \( x + 4y = 3 \)?

   Note: If two lines are perpendicular, there is a relationship between the coefficients of one equation and the coefficients of the second equation.
5. We can find equations of lines:

a) vertical lines: a line that is vertical and passes through the point (3, -2) →

b) horizontal lines: a line that is parallel to the x-axis and passes through the point (-1, 4) →

c) other lines (slant): passes through the point (2, -1) and is perpendicular to the line \( y = \frac{2}{3}x - 4 \)

Word: ( #5/131) Suppose a calculator manufacture has the total cost function \( C(x) = 17x + 3400 \) and the total revenue function \( R(x) = 34x \).

What is the equation of the profit function for this commodity? ______________

What is the profit on 50 items? ______ What is the marginal profit and what does it mean? ______

What is the profit from selling one more item if 50 are currently being sold? ___________

If you went through 1311, 1302, you saw some other uses of linear equations(functions): linear programming, inequalities.

Quadratic Functions:

What is the graph of the function represented by \( y = ax^2 + bx + c \)? ______________

Examples of the above type of curve

\( y = x^2 \) ==> ________________

\( y = 2x^2 - 4 \) ==> ________________

\( y = -4x^2 + 16 \) ==> ________________

\( y = -2x^2 - 8x + 5 \) ==> ________________

\( y^2 = 2x - 4 \) ==> ________________ Is this a function? Why or why not? ________
**Quadratic Functions:**

\[ y = Ax^2 + Bx + C \], if \( A \neq 0 \), then the graph is a parabola

1. If \( A > 0 \), then it opens upward
2. If \( A < 0 \), then it opens downward
3. What if \( A = 0 \)?

The vertex of the parabola is found at \( (\frac{-B}{2A}, f(\frac{-B}{2A})) \)

ex. graph \( f(x) = x^2 + 2x \) by finding the vertex, and the x and y–intercepts

ex. graph \( g(x) = -2x^2 + 5x - 3 \) (use the x and y-intercept)

---

Word examples:

1. (#39/165) The sensitivity \( S \) to a drug is related to the dosage \( x \) by \( S(x) = 1000x - x^2 \). What dosage gives the maximum sensitivity?

2. (#27/174)

\[
C(x) = 3600 + 25x + \frac{1}{2}x^2, \quad R(x) = (175 - \frac{1}{2}x)x \Rightarrow \text{find break even point, profit equation, compare: max profit with max revenue (same place?)}, \text{ compare: break-even points with zeros of } P(x)
\]
Other Types you should be familiar with

Absolute value:
\[ y = |x| \implies \ldots \quad y = |2x - 4| \implies \ldots \]
\[ |y| = x - 2 \implies \ldots \]

Polynomials:
\[ y = A_n x^n + A_{n-1} x^{n-1} + \ldots + A_1 x + A_0 \quad \text{degree} \ldots \quad \text{if} \quad A_n \neq 0, \ A_0 \text{is the constant term} \]

Examples of polynomials.
1) \[ f(x) = -3x + 2 \]
2) \[ P(x) = -4x^2 + 2 \]
3) \[ y = x^3 + 8 \]
4) \[ f(x) = x^4 + 5x^2 + 4 \]

Others may look and behave like polynomials in terms of algebraic properties but they are not polynomials.

ex. 1) \ldots (quotients) \quad 2) \ldots \quad \text{Stamp} \quad 3) \ldots \quad \text{another branch}
Some may not behave like polynomials

4) \( g(x) = \log \frac{\sin(x)}{1 - 2^x} \)

5) encounter later: normal curve

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

Branch Functions:

1) Absolute Value: \( y = | x - 3 | \)

2) inequality: \( f(x) = \)

\[
\begin{align*}
&2x + 3 \quad \text{if} \quad x \geq 2 \\
&x^2 - 1 \quad \text{if} \quad x < 2
\end{align*}
\]

3) equality: \( g(x) = \)

\[
\begin{align*}
&x + 5 \quad \text{if} \quad x = 2 \\
&2x \quad \text{if} \quad x \neq 2
\end{align*}
\]

Notation:

Find \( y \) when \( x = -7 \)  
Find \( f(x) \) when \( x = 3 \)  
Find \( g(x) \) when \( x = h \).
Some other useful functions and their graphs.

Exponential functions: \( f(x) = B^x \)

Word examples:

1) Invest $2000 at 4 ½ % per year compounded monthly, find the future value of this investment after 54 months.

\[ S = P \left( 1 + \frac{r}{k} \right)^n \]

2) page 384: \( y = w_0 e^{-h(t-t_0)} \)

This function models the exponential decay (the decrease in the number of atoms of a radioactive element.
\( w_0 \) is the number of atoms at time \( t_0 \) and \( h \) is a constant that depends on the element.

Graph \( f(x) = 2^x \), find \( x \) and \( y \) – intercepts

Graph \( h(x) = -3^x \)

I will assume that you have seen the basic construction of these functions.

Graph \( g(x) = 5^{-x} \), find \( g(0) = \), \( g(h) = \)
Answer Questions about exponential functions.

Is \( f(x) = 2^{x^2} \) a function? Why or why not? What is its x-intercept? y-intercept?

What is the domain of

\( y = 4^x \)? _________________

\( g(x) = 5^{-x} \) → _________________

What is the range of

\( f(x) = (1/4)^x \) → _________________

\( h(x) = -2^x \) → _________________
Logarithmic functions:  \( f(x) = \log_b x \)

Word Examples:  See page 405 in text

1) 51: Richter scale  
   \( R = \log \left( \frac{I}{I_o} \right) \)  
2) 55: Loudness (decibels)  
   \( L = 10 \log \left( \frac{I}{I_o} \right) \)

3) #59: pH levels  
   \( \text{pH} = -\log [H^+] \)

Graph  \( f(x) = \log_2 x \), include x and y-intercepts, find  \( f(0) = \),  \( f \left( \frac{1}{8} \right) = \)

Logarithms: exponents → given a value of x and a particular base, the logarithm changes x into an exponent.

\( \log_2 8 \) (the logarithm changes 8 into an exponent) →

\( \log_5 625 = \) 

\( \log_\frac{1}{3} 16 = \) 

Relationship between exponents and logarithms:

\( \log_b x = y \rightarrow b^y = x \)

for example:  \( \log_7 49 = 2 \)  Since →  \( 7^2 = 49 \)

\( \log_{10} 1000 \)
Graph of logarithmic functions:

\[ f(x) = \log_2 x \]

Note: Since the two most commonly used bases are 10 and \( e \) we have the following notation

Recall: \( \sqrt[5]{5} \) is generally written as \( \sqrt{5} \) → we usually leave out the index and understand that it is implied to be there

1) \( \log_{10} x \) is generally written as \( \log x \) → base is ten is understood to be there. Almost all other bases must be written. We call this a common logarithm.

2) the one other case in which the base is generally not written is when we use base \( e \)

\[ \log_e x: \text{ we usually write } \log_e x = \ln x, \text{ we call this the natural logarithm of } x. \]

Logarithmic functions.

Domain:

\[ f(x) = \log_3 x \rightarrow \text{_________} \quad g(x) = \log_4 (x - 2) \rightarrow \text{_________} \]

Range:

\[ f(x) = \log_3 x \rightarrow \text{_________} \]


Properties:

a) $\log_b x$ : these are defined for $b > 0$ and $x > 0$

b) $\log_b b = \underline{\text{__________}}$

c) $\log_b 1 = \underline{\text{__________}}$

d) $\log_b b^x = \underline{\text{__________}}$

e) $\log_b (xy) = \log_b x + \log_b y$

f) $\log_b (x/y) = \log_b x - \log_b y$

g) $\log_b x^k = k \log_b x$

More examples and questions about lines:

What is the slope of

a) a horizontal line $\Rightarrow \underline{\text{__________________}}$

b) a vertical line $\Rightarrow \underline{\text{__________________}}$

c) a line that passes through the points A (2, -2) and B (4, -3) $\Rightarrow \underline{\text{__________}}$

d) line that is represented by $y = -3x - 2$

e) What about $2x - 3y = 4$
Average Rate of Change

f) the secant line that passes through \( f(-2) \) and \( f(1) \) if \( f(x) = 2x^2 - x + 1 \). This last slope represents what we call the average rate of change.

: goat – wanders around all day – how far do you have to go to pick it up. even though the goat traveled a lot longer than you – as far as you are interested the distance corresponds only from the beginning point to the final point.

: another example – stock market during the day, at end of day, change in value

g) Find the average rate of change, \( A_{rc} \) of \( g(x) = -2x^3 + 12 \) as \( x \) changes from 0 to 4.

Why do we need the following idea?

examples – pollution

The function \( C(p) \) represents the cost of removing \( p \) percent of the particular pollution from the emissions at a given plant (factory).

\[
C(p) = \frac{7300p}{100 - p}
\]

We are able to answer questions such as

What would be the cost in the example above to remove 25% of the pollution? __________

What about 75%? __________

How would you answer the question for 100%? Could we answer the question? __________

Other examples on page 639 and 640.
Limits:
the question has to do with values of x (domain), the answer has to do with values of y (the range)

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>4.9</th>
<th>4.99</th>
<th>4.999</th>
<th>4.9999</th>
<th>4.99999</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>8</td>
<td>9</td>
<td>9.1</td>
<td>9.9</td>
<td>9.99</td>
<td>9.999</td>
<td>9.9999</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>9</th>
<th>6</th>
<th>5.1</th>
<th>5.01</th>
<th>5.001</th>
<th>5.0001</th>
<th>5.00001</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>7</td>
<td>6</td>
<td>5.01</td>
<td>5.001</td>
<td>5.0001</td>
<td>5.00001</td>
<td>5.000001</td>
</tr>
</tbody>
</table>

need to look over these tables

If I asked you what is \( f(2) \), what would you answer? \[ \_\_\_\_\_\_ \] what about \( f(9) \)? \[ \_\_\_\_\_\_ \] \( f(5) = \_\_\_\_\_\_ \)

If I asked you what is happening to the values of y as x gets closer and closer to 5, what would you say?

Limits:
Let \( f(x) \) be a function defined on some interval around \( x = c \).

We write \( \lim_{x \to c} f(x) = L \) and say “the limit of \( f(x) \) as \( x \) approaches \( c \) is equal to \( L \)”

to mean the closer that \( x \) gets to \( c \), the closer \( f(x) \) gets to \( L \). We do not need to know what happens at \( x = c \).

When look at values of the limit, we are looking at the y-coordinates.

We have directional limits,
from the left: \( \lim_{x \to c^-} f(x) \)
from the right: \( \lim_{x \to c^+} f(x) \)

and from both directions: \( \lim_{x \to c} f(x) \). This limit exists only if the left and right limits exist and are equal to each other.
Ex. Let find all three limits of $f(x) = 4x - 1$ if $c = 2$

a) 

b) 

c) 

Ex. Do the same thing for each of the following functions.

1) $f(x) = x^2$, $c = -1$

2) $f(x) = -4$ if $c = 2$

Ex. Look at the following graphs. Do as above with $c = 0$ and with $c = 4$

a) 

b)
Properties of limits

1. \( \lim_{x \to c} k = k \)  
   ex. \( f(x) = 3 \implies \) __________  
   ex. \( y = \frac{1}{2} \implies \) __________

2. \( \lim_{x \to c} x = c \)  
   ex. \( f(x) = x \implies \) __________  
   ex. \( y = x \implies \) __________

3. \( \lim [f(x)g(x)] = \lim f(x) \cdot \lim g(x) \)  
   ex. \( f(x) = 2x \implies \) __________
   ex. \( y = -4x \implies \) __________

4. \( \lim [f(x) + g(x)] = \lim f(x) + \lim g(x) \)  
   ex. \( f(x) = 3 + x \implies \) ______________
   ex. \( y = -2 + x \implies \) ______________
   ex. \( h(x) = 4 - 2x \implies \) ______________

5. \( \lim \left[ \frac{f(x)}{g(x)} \right] = \frac{\lim f(x)}{\lim g(x)}, \text{ if } \lim g(x) \neq 0 \)  
   ex. \( f(x) = \frac{2x}{x - 3} \implies \) ______________
   ex. \( y = \frac{3 + x}{x - 4} \implies \) ______________

6. \( \lim k f(x) = k \lim f(x) \)  
   ex. \( f(x) = 6x \implies \) __________  
   y = -5x \implies \) __________

7. \( \lim x^k = [\lim x]^k \)  
   ex. \( f(x) = x^2 \implies \) __________  
   y = x^3 \implies \) __________
Because of all of these properties we can say the following

If \( P(x) \) is a polynomial, then
\[
\lim_{x \to c} P(x) = P(c)
\]

Examples of limits -

1. \( \lim_{x \to 3} 5 = _________ \)
2. \( \lim_{x \to -2} (2x - 1) = _________ \)

2. \( \lim_{x \to 3} \frac{2}{x} = _________ \)
3. \( \lim_{x \to -4} \frac{x}{x + 4} = _________ \)

4. \( \lim_{x \to 2} \frac{x - 2}{x^2 - 4} = _________ \)
5. \( \lim_{x \to -2} \frac{2}{x - 2} = _________ \)

\( f(x) = \begin{cases} 
2x - 1 & \text{if } x \geq 2 \\
3x & \text{if } x < 2 
\end{cases} \)

\( g(x) = \begin{cases} 
2x & \text{if } x < 1 \\
x^2 + 1 & \text{if } x \geq 1 
\end{cases} \)

\( h(x) = \begin{cases} 
2 & \text{if } x > 1 \\
0 & \text{if } x = 1 \\
4x & \text{if } x < 1 
\end{cases} \)

Before we answer questions about limits let’s see if we understand functional values.

\( f(2) = _________ \)
\( f(-3) = _________ \)
\( g(1) = _________ \)

\( g(2) = _________ \)
\( h(1) = _________ \)
\( h(2) = _________ \)

\( h(-1) = _________ \)
\[
\begin{align*}
\text{f}(x) &= \begin{cases} 
2x - 1 & \text{if } x \geq 2 \\
x^3 & \text{if } x < 2
\end{cases} \\
\text{g}(x) &= \begin{cases} 
2x & \text{if } x < 1 \\
x^2 + 1 & \text{if } x \geq 1
\end{cases} \\
\text{h}(x) &= \begin{cases} 
2 & \text{if } x > 1 \\
0 & \text{if } x = 1 \\
4x & \text{if } x < 1
\end{cases}
\end{align*}
\]

7. \( \lim_{{x \to 2^+}} \text{f}(x) = \) \\
8. \( \lim_{{x \to 2^-}} \text{f}(x) = \)

9. \( \lim_{{x \to 1^+}} \text{g}(x) = \) \\
10. \( \lim_{{x \to 3^-}} \text{g}(x) = \)

11. \( \lim_{{x \to 1^+}} \text{h}(x) = \) \\
12. \( \lim_{{x \to 1^-}} \text{h}(x) = \)

13. \( \lim_{{x \to -1}} \text{h}(x) = \)
Infinite Limits and Limits at infinity

example:

\[ \lim_{x \to -1} \frac{x + 1}{x^2 - 1} = \ldots \]

\[ \lim_{x \to 2} \frac{x + 2}{x^2 - 4} = \ldots \]

\[ \lim_{x \to \infty} \frac{30000}{x} = \ldots \]

Properties of limits at infinity.

1. \( \lim_{x \to \infty} K = \ldots \), where \( K \) is a constant

2. \( \lim_{x \to \infty} x = \ldots \)

3. \( \lim_{x \to \infty} \frac{1}{x} = \ldots \)

4. \( \lim_{x \to \infty} \frac{1}{x^k} = \ldots \)

What about previous properties of limits

ex. \( \lim_{x \to \infty} x^2 + 2x - 4 = \ldots \)

ex. \( \lim_{x \to \infty} \frac{2}{x^2 + 2x - 4} = \ldots \)

ex. \( \lim_{x \to \infty} \frac{2}{x - 2} = \ldots \)

ex. \( \lim_{x \to \infty} \frac{x + 2}{x - 2} = \ldots \)
Undefined Limits and infinite Limits (Graphs)

Use the given graph to answer the questions that follow.

a) \( f( ) = \) ________  
b) \( f( ) = \) ______________  
c) \( f( ) = \) _______

d) \( \lim_{x \to } f(x) = \) ________  
e) 

f) 
g)
More examples of Limits

ex. \( \lim_{x \to \infty} 2x = \) \_

\( \lim_{x \to \infty} x^2 = \) \_

\( \lim_{x \to \infty} \frac{1}{x} = \) \_

ex. \( \lim_{x \to \infty} 4 = \) \_

\( \lim_{x \to \infty} (2x - 4) = \) \_

\( \lim_{x \to \infty} \frac{2x + 4}{100000} = \) \_

ex. \( \lim_{x \to \infty} \frac{2x}{3x + 1} = \) \_

\( \lim_{x \to \infty} \frac{x^2 + 5x}{x^3 - 2x + 1} = \) \_

ex. \( \lim_{x \to \infty} \frac{x^3 + 1}{x + 1} = \) \_


Main ideas of limits

\[
\begin{align*}
\lim_{x \to c} k &= \_\_\_\_\_ & \lim_{x \to z} k &= \_\_\_\_\_ & \lim_{x \to c} x &= \_\_\_\_\_ & \lim_{x \to z} x &= \_\_\_\_\_ \\
\lim_{x \to \infty} P(x) &= \_\_\_\_\_ & \lim_{x \to \infty} P(x) &= \_\_\_\_\_ \\
\lim_{x \to c} \frac{P(x)}{Q(x)} &= \_\_\_\_\_ & \lim_{x \to \infty} \frac{P(x)}{Q(x)} &= \_\_\_\_\_ \\
\lim_{x \to c} f(x) &= \_\_\_\_\_ & \lim_{x \to z} f(x) &= \_\_\_\_\_ \\
\end{align*}
\]

Let

\[
\begin{align*}
f(x) &= \begin{cases} 
2x & \text{if } x \leq 1 \\
x^2 + 1 & \text{if } x > 1 
\end{cases} & \quad g(x) &= \begin{cases} 
2 & \text{if } x \neq 5 \\
-4x & \text{if } x = 5 
\end{cases} \\
h(x) &= 3 & \quad t(x) &= \begin{cases} 
x + 2 & \text{if } x \geq 0 \\
x^2 & \text{if } x < 0 
\end{cases} 
\end{align*}
\]

Find each of the following limits.

1) \( \lim_{x \to -2} h(x) = \_\_\_\_\_ \)

2) \( \lim_{x \to \frac{1}{2}} 12h = \_\_\_\_\_ \)

3) \( \lim_{x \to 4} \frac{x}{x - 3} = \_\_\_\_\_ \)

4) \( \lim_{x \to 2} \frac{3}{x - 2} = \_\_\_\_\_ \)

5) \( \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \_\_\_\_\_ \)

6) \( \lim_{x \to 1} f(x) = \_\_\_\_\_ \)

7) \( \lim_{x \to 1.4} f(x) = \_\_\_\_\_ \)

8) \( \lim_{x \to 5} g(x) = \_\_\_\_\_ \)

9) \( \lim_{x \to -4} t(x) = \_\_\_\_\_ \)

10) \( \lim_{x \to 0} t(x) = \_\_\_\_\_ \)

11) \( \lim_{x \to \infty} 3 = \_\_\_\_\_ \)

12) \( \lim_{x \to \infty} x = \_\_\_\_\_ \)
Continuity:

Def. A function f(x) is said to be continuous at x = c provided the following are satisfied:

1) f(c) exists --- there is an actual y-value at x = c, c is part of the domain of f(x)

2) \[ \lim_{x \to c} f(x) \] exists, left and right limit approach the same value

3) f(c) = \[ \lim_{x \to c} f(x) \] – the answer for #1 is the same as the answer for #2.

If a function is not continuous at x = c, then we say that f is discontinuous and x = c is a point of discontinuity.

If a function is continuous at every single value of x, the we say that f is continuous.

ex. Show f(x) = 2x – 1 is continuous at x = 4.

1) 
2) 
3)

Show whether f(x) is continuous or discontinuous at x = 2

a) f(x) = \[ \frac{x^2 - 4}{x - 2} \]

b) f(x) = \[ 2x \text{ if } x > 2 \]

c) f(x) = \[ x - 2 \text{ if } x \neq 2 \]

d) f(x) = \[ x + 2 \text{ if } x > 2 \]

c) f(x) = \[ x^2 \text{ if } x = 2 \]

d) f(x) = \[ 2x - 2 \text{ if } x \leq 2 \]

Note: If a function is not continuous at x = c, we say f is discontinuous – discontinuous at c.

If a function is continuous at each x = c, we say f is continuous--- continuous at every x.
ex. Use the given graph to find all points at which the limit is undefined (DNE). Find all the points at which the curve is discontinuous.

Find the points of discontinuity of the following functions.

a) \( f(x) = \frac{3}{x - 3} \)

b) \( f(x) = \begin{cases} 2x & \text{if } x > 3 \\ 6 & \text{if } x \leq 3 \end{cases} \)

c) \( f(x) = -2 \)

d) \( f(x) = \frac{x - 2}{4} \)

e) \( y = \frac{3x}{x^2 - 9} \)

Recall the definition of the slope of a line - \( m = \) 

Since we write \( f(x) \) to mean the value of \( y \) at \( x \) - Replace \( y_2 \) with \( f(x_2) \) and \( y_1 \) with \( f(x_1) \). We can write

\[ m = \]

ex. If we want to find the slope of the line \( 2x - y = 8 \) by using the \( y \) values at \( x = -1 \) and \( x = 3 \), we get

\[ m = \]
We have defined this value before as the average rate of change of \( f \), \( A_{rc} \), as \( x \) changes from \( x_1 \) to \( x_2 \).

ex. Find \( A_{rc} \) of \( f(x) = 2x^3 - 1 \) as \( x \) changes from -1 to 2.

ex. Find \( A_{rc} \) of \( g(x) = -x + 2 \) as \( x \) changes from -1 to 2.

What if started to select values of \( x_2 \) closer and closer to \( x_1 \) ? What kind of slope would we get? Would it be the slope of a secant line or something else?

![Graph](image)

**Def.** Let \( f(x) \) be a continuous function at \( x = c \). We define the instantaneous rate of change, \( I_{rc} \), by

\[
I_{rc} = \lim_{{h \to 0}} \frac{f(c + h) - f(c)}{h}
\]

\( I_{rc} \): the slope of the tangent line at a given value of \( x \), at \( x = c \)

\( A_{rc} \): the slope of a secant line at two values of \( x \), \( x_1 \) and \( x_2 \).

ex. Find the \( I_{rc} \) at \( x = 1 \) in each of the following functions.

a) \( f(x) = 4 \)  
b) \( f(x) = -2x - 5 \)

b) \( f(x) = x^2 - 1 \)

What about at \( x = 2, -4, 7, \pi, \ldots \)? In each case we have to redo the work above. Time consuming!

The following is an alternate approach if we have to find more than one instantaneous rate of change.
Def. Let \( f \) be a continuous function on some interval \((a,b)\). We define the derivative of \( f \), written \( f' \), \( f'(x) \), \( dy/dx \), ... by

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

It looks like the \( I_{rc} \) except that it is not at one value \( x = c \) but at any value (at all values of \( x \)).

1) _____________________________ refers to the slope of a secant line to a curve at two points.

2) _____________________________ refers to the slope of a tangent line at a point

3) _____________________________ refers to the slope of a tangent line at each point of a curve

ex. Find \( A_{rc} \) of \( f(x) = x^3 - 1 \) as \( x \) goes from 1 to 3

ex. Find \( I_{rc} \) of \( f(x) = x^3 - 1 \) at \( x = 1 \)

ex. Find \( f'(x) \) if \( f(x) = x^2 + 1 \) What is \( f(1) \)? ________. \( f'(1) \)? _______

Because the derivative is defined as a limit – some properties of limits carry over to derivatives.
Properties of the derivative of a function.

1) if \( f(x) = k \) where \( k \) is a constant, then \( f'(x) = \) ___________
*********** the derivative of a constant is zero *******

ex. \( f(x) = -4 \) ==> _______________

2) if \( f(x) = x \), then \( f'(x) = \) ___________
*********** the derivative of \( x \) is always one ******

ex. \( f(x) = x \)

2.5) if \( f(x) = kx \) where \( k \) is a constant, then \( f'(x) = \) ___________

ex. \( f(x) = 4x \) ==> _______________

3) if \( f(x) = g(x) + h(x) \), then \( f'(x) = \) ___________
*********** the derivative of a sum of two functions is always the sum of the derivatives*******

ex. \( f(x) = x + 7 \) ==> _______________

ex. \( f(x) = 3x + 2 \) ==> _______________

ex. \( g(x) = -2x + 5 \) ==> _______________

ex. \( h(x) = -x - 2 \)

4) if \( f(x) = x^k \), then \( f'(x) = \) ___________

ex. \( y = x^4 \) ==> _______________

ex. \( y = x^{20} \) ==> _______________

ex. \( y = -x^{7/2} \) ==> _______________

ex. \( y = x^{-4} \) ==> _______________

ex. \( f(x) = \sqrt[3]{x} \) ==> _______________
5) Product: if $f(x) = g(x) \cdot h(x)$, then $f'(x) = g(x) \cdot h'(x) + g'(x) \cdot h(x)$

"the derivative of a product is equal to -- the 1st times the derivative of the 2nd plus the 2nd times the derivative of the 1st"

ex. $f(x) = 5x$ $\rightarrow$ $f'(x) =$

ex. $g(x) = \frac{2}{3}x$ $\rightarrow$ $g'(x) =$

ex. $y = (2x - 3)(5x + 3)$ $\Rightarrow$ $y' =$

ex. $f(x) = x^4(2x^2 - 5x + 2)$ $\Rightarrow$ $f'(x) =$

6) Quotient: if $f(x) = \frac{g(x)}{h(x)}$, then $f'(x) = \frac{h(x) \cdot g'(x) - g(x) \cdot h'(x)}{h^2(x)}$

"the derivative of a quotient --- Numerator: the bottom \cdot der. of top minus the top \cdot der. of bottom
Denominator: the original denominator squared"

ex. $y = \frac{2x - 3}{x + 4}$

ex. $f(x) = \frac{1 - 3x}{2x - 1}$

7. $y = [f(x)^k]$, $y = k[f(x)^{k-1}] \cdot f'(x)$

ex. $y = (2x - 4)^3$ $\Rightarrow$ $y' =$

ex. $u = (1 - 2x^3)^3$ $\Rightarrow$ $u' =$

ex. $t = (r^4 - 2)^2$ $\Rightarrow$
Chain Rule and the Power Rule
The last rule discussed above is part of what is called the chain rule.

ex. We know that if \( y = 2x^2 \implies y' = \) _________________

What about a function like \( y = (2x^2 + 1)^2 \)? _________________

First take the function \( y = u^w \) -- find the derivative _________________ Find the derivative of \( u = 2x^2 + 1 \)

What do these two results have to do with the question above?

ex. Let \( f(x) = (1 - 4x)^4 \implies f'(x) = \) _________________

Examples of derivatives
ex. find the derivative of each of the following functions

1) \( f(x) = -4 \implies \) _________________ 2) \( f(x) = -3x \implies \) _________________

3) \( f(x) = 4x - 7 \implies \) _________________ 4) \( y = x^6 \implies \) _________________

5) \( g(x) = -4x^{-4} \implies \) _________________

6) \( h(x) = (2x^2 - 2)(5x - x^2) \implies \) _________________

7) \( y = \frac{3u - 4}{2 - 3u} \implies \) _________________

8) \( v = (2u - 3)^4 \implies \) _________________
Sets:
Definition: group of objects that have a well-defined property in common

set of letters of the English alphabet: \{ \ a, b, c, ..., x, y, z \}  set of all even prime numbers: \{ 2 \}
\{ x \mid x \text{ is an integer with } x^2 < 9 \},

Elements: any object in the set – use the symbol \( \in \)

\[ 2 \in \{ 1, 2, 3 \} \quad \text{while} \quad 4 \not\in \{ 1, 2, 3 \} \]

Subsets: any collection of the given set -- use the symbol \( \subseteq \) or \( \subset \) (we will usually use the first)

\{ 1, 2 \} \subseteq \{ 1, 2, 3, .. \}    \text{the set of natural numbers is a subset of the set of integers}

Sets of Real Numbers: a number that can be written in decimal form

Real Numbers

Rational Numbers
( can be written as a fraction)

Integers

Whole numbers

Negative Integers
\{ ..., -3, -2, -1 \}

Simple Fractions
-1/3, 5/8, 1 2/3, 17/15

Zero
\{ 0 \}

Natural numbers
\{ 1, 2, 3, ... \}

One
\{ 1 \}

Prime
\{ 2, 3, 5, 7, 11, ... \}

Composite
\{ 4, 6, 9, 10, ... \}

Absolute Value: distance from the origin

\[ |c| \geq 0, \text{ never negative} \]

\[ | -2/3 | = 2/3 \]
\[ | x^2 | = x^2 \]
\[ |x| = x \text{ if } x \text{ is a whole number} \]
\[ |x| = -x \text{ if } x \text{ is a neg. integer} \]
Relations – VS- Functions

A function is always a relation. A relation may or may not be a function.

sets:

graphs

equations

Domain: \( (x's) \)
Range: \( (y's) \)

Special Functions:

\[ y = 2^x \]
\[ y = \log_4 x \]
\[ y = |2 - 4x| \]
\[ y = \frac{x + 2}{x^2 + 5x + 6} \]

Limits:

left limit  right limit  both  infinite limits  at infinity

Continuity:
Average rate of change ( \( A_{rc} \) )

Instantaneous rate of change ( \( I_{rc} \) )

Derivatives (\( f'(x) \) )
Higher order derivatives

ex. $y = 4x^3 - 2x + 7$ find the derivative of this function

ex. What about the derivative of $f(x) = x^4 - x^2 + 7x - 2$

Can you take the derivative of the derivative?

What about the derivative of the derivative of the derivative?

One place of interest:

1$^{st}$ derivative: provides information about function as to whether it is increasing, decreasing, or possibly neither. We may have an interest in finding where the derivative is zero.

2$^{nd}$ derivative: provides information that can be useful to find inflection points, concavity of a function.
Now we can put all this together and use it to get a rough sketch of the graph of functions like

We will use x and y-intercepts to help us

1) \( f(x) = x^2 - 4x \)  
2) \( g(x) = 1 - 2x^2 \)  
3) \( h(x) = x^3 - 12x + 1 \)

Use cost functions and derivatives to give an estimate of the approximate cost to produce the \( x + 1 \)st item
Additional Material
(and review)

Find the area under each of the following curves

\[ f(x) = 2 \quad \text{and the x-axis from } x = 0 \text{ to } x = 4 \]

\[ g(x) = 2x \quad \text{from } x = 0 \text{ to } x = 4 \]

\[ h(x) = 2x + 3 \quad \text{from } x = 0 \text{ to } x = 4 \]

Look at these functions from a different viewpoint

\[ F(x) = 2 \quad G(x) = 2x \quad H(x) = 2x + 3 \]

We have an idea on how to find the area under a curve -

Indefinite Integrals

\[ \int 5 \, dx = \quad \int 5x \, dx = \]

\[ \int_1^3 5 \, dx = \quad \int_1^3 \frac{1}{2}x \, dx = \]
What about
\[ \int_{-1}^{2} (3x^2 - 2x + 3) \, dx \]

**Brief Review:**

1. The derivative measures: _______________________________________________________

2. ex. Find the slope of the line tangent to the curve \( f(x) = 3x^2 + 2x - 3 \) at \( x = 2 \) \( \rightarrow \) ____________

   ex. What is the equation of the tangent line to \( g(x) = x^2 - 4x + 2 \) at \( x = 2 \) _________________

   ex. If \( C(x) = \frac{2x + 3}{x - 2} \) represents the cost in producing \( x \) items, then

   what is the marginal cost at \( x = 3 \) ? ______________

   Give me an estimate of the cost of producing the 4\(^{th}\) item. ___________

   Is there any particular value of \( x \) (production) that we should be careful with when calculating the cost? ______

   What is the exact cost to produce the 4\(^{th}\) item? ____________
3. Find the critical values of the function \( g(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x - 1 \)

Do they represent relative (local) maximums or relative (local) minimums? ________________

Where are their inflection points? ______________

Rough but accurate graph without the use of a graphing calculator.

Where is the function increasing? Where is it decreasing?

Where is it concave up? Where is it concave down?

4. Integral.

Given \( F(x) = f'(x) = 2x - 4 \), then what was \( f(x) \)?

If \( F(x) = 3x^2 - 2x + 3 \), then find the area under the curve between 0 and 2.
Special Terms and Definitions and Integral Calculus:

1. Find each of the following
   a. Find the indefinite integral
      \[ \int (1 - x) \, dx = \] ____________________
   
   b. Find the definite integral
      \[ \int_{-1}^{2} (3x^2 - 2x + 3) \, dx \]

2. If the derivative of \( f(x) \) has to do with slopes of tangent lines to the curve, what does \( \int f(x) \) have to do with the curve?

3. Use the functions \( f(x) = 2x^3 - 12x + 1 \)
   to find
      a) all critical values

      b) all inflection points

      c) where the function is increasing

      d) where is the function concave down
4. Use the function \( f(x) = 3x^3 - 2x + 1 \) to find
   
   a) \( f(1) = \) _____________  
   b) \( f'(1) = \) _____________  
   c) \( f''(1) = \) _____________

5. Find the equation of the line tangent to \( f(x) = x^2 + 2x + 1 \) at \( x = 2 \).

6. Sketch the curve in #3

   ![Graph of the curve](image)

7. Find the marginal profit of the product whose cost is given by \( C(x) = \) and revenue equation is represented by \( R(x) = \)

   When will the profit be at a maximum?

   What is the profit to produce the first 10 items? ______________

   What is the estimated profit to produce the 11\(^{th}\) item? ______________  the exact profit? ______________
8. Find the derivative of

\( y = \frac{2 - x^2}{3} \), then

\( y' = 3(2 - x^2)^2 \cdot (-2x) = -6x(2 - x^2)^2 \)

a) \( y = e^x \implies \) __________________

b) \( y = e^{2x} \implies \) ________________

c) \( y = e^x \implies \) ________________

d) \( y = \ln x \implies \) ________________

e) \( y = \ln x^2 \implies \) ________________

f) \( y = \ln 2x \implies \) ________________

9. True or False

__________ a) all continuous functions are differentiable

__________ b) all differentiable functions are continuous

__________ c) a function that is differentiable everywhere has a limit everywhere

__________ d) a function that has a limit everywhere is continuous everywhere

__________ e) the area under any normal curve (normal distribution) is equal to 1 (square) unit

__________ f) a standard normal curve has a mean of 1

__________ g) a binomial r.v. only has whole number values from 0 to n

__________ h) a variance of a r.v. is never negative.
1. Find the following limits from the given tables

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>4.9</th>
<th>4.99</th>
<th>4.999</th>
<th>4.9999</th>
<th>4.99999</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>8</td>
<td>9</td>
<td>9.1</td>
<td>9.9</td>
<td>9.99</td>
<td>9.999</td>
<td>9.9999</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>9</th>
<th>6</th>
<th>5.1</th>
<th>5.01</th>
<th>5.001</th>
<th>5.0001</th>
<th>5.00001</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>7</td>
<td>6</td>
<td>5.01</td>
<td>5.001</td>
<td>5.0001</td>
<td>5.00001</td>
<td>5.000001</td>
</tr>
</tbody>
</table>

a) Find

\[
\lim_{x \to 5^+} f(x) = \text{__________}
\]

b) Find

\[
\lim_{x \to 5^-} f(x) = \text{__________}
\]

c) \[
\lim_{x \to 5} f(x) = \text{__________}
\]

2. Find each of the following limits

a) \[
\lim_{x \to 2} 12 = \text{__________}
\]

b) \[
\lim_{x \to -2} x = \text{__________}
\]

c) \[
\lim_{x \to 3} 4x - 2 = \text{__________}
\]

d) \[
\lim_{x \to 2} f(x) = \text{__________}, \text{ if } f(x) = x^2 + 2x - 1
\]

3. Find the following limits. Use the graph.

a) \[
\lim_{x \to 2} f(x) = \text{__________}
\]

b) \[
\lim_{x \to -1} f(x) = \text{__________}
\]

c) \[
\lim_{x \to 5} f(x) = \text{__________}
\]
1. Find the slope of the line that passes through the points A(2, -4) and B(-4, -8). \( m = \) ________________

2. What is the slope of the horizontal line that passes through the point P(-4, 1)? \( m = \) ________________

3. Find the slope of the line parallel to the y-axis passing through the point (1, -2). \( m = \) ________________

4. What is the slope of the secant line to the curve \( f(x) = x^2 + 2x \) from \( x = 2 \) to \( x = 5 \)? \( m = \) ________________

5. Find each of the following limits.
   \( \lim_{x \to 2} \frac{2x}{x} = \) _______  \( \lim_{x \to -1} \frac{-4}{x} = \) __________
   \( \lim_{x \to 2} \frac{2 - x - x^2}{x} = \) __________  \( \lim_{x \to -1} \frac{2}{x - 1} = \) __________
   \( \lim_{x \to 2} \frac{x - 2}{x^2 - 4} = \) __________  \( \lim_{x \to -1} x^2 = \) __________

6. Find the limit \( f(x) = \) ________________, if \( f(x) = \) \( x \) if \( x < 3 \)
   \( x \to 4 \) \( x^2 \) if \( x > 3 \)

7. Find the following limits. Use \( f(x) \) from #6
   a) \( \lim_{x \to 2} f(x) = \) _______
   b) \( \lim_{x \to 2^+} f(x) = \) _______
   c) \( \lim_{x \to 2} f(x) = \) _______

8. \( \lim_{x \to \infty} 4 = \) __________  \( \lim_{x \to \infty} (2x - 4) = \) __________  \( \lim_{x \to \infty} \frac{2x + 4}{100000} = \) __________

9. \( \lim_{x \to \infty} \frac{2x}{3x + 1} = \) __________  \( \lim_{x \to \infty} \frac{x^2 + 5x}{x^3 - 2x + 1} = \) __________
10. Use the given graph to find each of the following values.

a) 

b) 

c) 

d) 

e) 

f) 

g)
1. Find all points where the limit of \( f(x) \) does not exist. 

2. Show \( f(x) = x^2 + 2 \) is continuous at \( x = 4 \)

3. Find one point of discontinuity of \( f(x) = \frac{x}{x^2 - 9} \)

4. Find the \( Ar_c \) of \( f(x) = x^2 - 4x \) as \( x \) changes from \( x = 0 \) to \( x = 3 \)
1. Find $A_{rc}$ of $f(x) = x^2 + 1$ as $x$ goes from -1 to 2.

2. Find the instantaneous rate of change of $f(x) = 1 - 4x$ at $x = 3$.

3. Complete the following:
   We say $f$ is continuous at $x = c$ provided
   
   1) 
   
   2) 
   
   3) 

4. What are the points of discontinuity of each of the following functions.
   
   a) 
   
   b) 
   
   c)
1. Find the instantaneous rate of change, \( I_x \) of \( f(x) = 1 - 2x^2 \) at \( x = -1 \)

2. Find all points (x’s) of discontinuity of each of the following functions. If continuous everywhere, then indicate so.
   a) \( y = \frac{x - 1}{4} \)
   b) \( f(x) = \frac{x + 1}{x^2 - 9} \)
   c) \( h(x) = \begin{cases} 2x & \text{if } x > 1 \\ x^2 + 1 & \text{if } x < 1 \end{cases} \)

3. Find \( h(1) \) _________ and \( h(2) \) _________ in the function above

3. Find the derivative of each of the following functions
   a) \( f(x) = -2x^3 + x - 2 \implies f'(x) = \) ________________
   b) \( g(x) = -2 \implies g'(x) = \) ______
   c) \( f(x) = -2x^{-4} \implies f'(x) = \) ________________
   d) \( y = (x + 1)^2 \implies y' = \) ________________
1. Find the instantaneous rate of change, \( I \) of \( f(x) = 1 - 2x^2 \) at \( x = -1 \)

2. Find all points (x’s) of discontinuity of each of the following functions. If continuous everywhere, then indicate so.
   a) \( y = \frac{x - 1}{4} \)
   b) \( f(x) = \frac{x + 1}{x^2 - 9} \)
   c) \( h(x) = \begin{cases} 
   2x & \text{if } x > 1 \\
   x^2 + 1 & \text{if } x < 1 
   \end{cases} \)

3. Find \( h(1) \) _________ and \( h(2) \) _________ in the function above

3. Find the derivative of each of the following functions
   a) \( f(x) = -2x^3 + x - 2 \Longrightarrow f'(x) = \) ________________
   b) \( g(x) = -2 \Longrightarrow g'(x) = \) ______
   c) \( f(x) = -2x^{-4} \Longrightarrow f'(x) = \) ________________
   d) \( y = (x + 1)^2 \Longrightarrow y' = \) ________________
Math 1312 – Week 15 – Quiz

1. Find each of the following
   a. Find the indefinite integral
   \[ \int (1 - x) \, dx = \ldots \]
   b. Find the definite integral
   \[ \int_{-1}^{2} (3x^2 - 2x + 3) \, dx \]

2. If the derivative of \( f'(x) \) has to do with slopes of tangent lines to the curve, what does \( \int f(x) \) have to do with the curve?

3. Use the functions \( f(x) = 2x^3 - 12x + 1 \) to find
   a) all critical values
   b) all inflection points
   c) where the function is increasing
   d) where is the function concave down

4. Use the function \( f(x) = 3x^3 - 2x + 1 \) to find
   a) \( f(1) = \ldots \)
   b) \( f'(1) = \ldots \)
   c) \( f''(1) = \ldots \)
5. Find the equation of the line tangent to \( f(x) = x^2 + 2x + 1 \) at \( x = 2 \).

6. Sketch the curve in #3

7. Find the marginal profit of the product whose cost is given by \( C(x) = \) and revenue equation is represented by \( R(x) = \)

When will the profit be at a maximum?

What is the profit to produce the first 10 items? ____________

What is the estimated profit to produce the 11\(^{th}\) item? ____________ the exact profit? ____________
8. Find the derivative of

(remember that if \( y = (2 - x^2)^3 \), then \( y' = 3(2 - x^2)^2 \cdot (-2x) = -6x(2 - x^2)^2 \)

a) \( y = e^x \Rightarrow \) __________________

b) \( y = e^{2x} \Rightarrow \) __________________

c) \( y = e^2 \Rightarrow \) __________________

d) \( y = \ln x \Rightarrow \) __________________

e) \( y = \ln x^2 \Rightarrow \) __________________

f) \( y = \ln 2x \Rightarrow \) __________________

9. True or False

__________ a) all continuous functions are differentiable

__________ b) all differentiable functions are continuous

__________ c) a function that is differentiable everywhere has a limit everywhere

__________ d) a function that has a limit everywhere is continuous everywhere

__________ e) the area under any normal curve (normal distribution) is equal to 1 (square) unit

__________ f) a standard normal curve has a mean of 1

__________ g) a binomial r.v. only has whole number values from 0 to n

__________ h) a variance of a r.v. is never negative.
Math 1312- QZ

1. Find all points where the limit of \( f(x) \) does not exist. _________________________________

2. Show \( f(x) = x^2 + 2 \) is continuous at \( x = 4 \)

3. Find one point of discontinuity of \( f(x) = \frac{x}{x^2 - 9} \)

4. Find the \( A_{eq} \) of \( f(x) = x^2 - 4x \) as \( x \) changes from \( x = 0 \) to \( x = 3 \)
1. Draw a normal curve with mean = 100 and standard deviation = 2

   Find the area to the right of 95 under the curve. ________________

2. A binomial r.v. X has n = 100 and p = 0.1. Find the expected value = _______ and the
   the standard deviation __________

   Use a normal curve to estimate the probability that X < 8. ________________

3. A weather forecaster will forecast the weather for the next year (365). During his 20 years
   of experience he has been right 25% of the time. Assuming that the same conditions will
   exist during the next year what is the probability that he will be right more than 90 days of
   the year?

   Use a normal curve to estimate your answer.
1. Let \( P(x) = 2x^4 - 4x + 2 \) be a given polynomial. Find
   a) \( P(0) = \) ___________
   b) \( P(-1) = \) ______________
   c) \( P(c) = \) ___________

2. Let \( f(x) = -x + 3 \). Find
   a) \( f(-2) = \) __________
   b) \( f(-h) = \) ______________
   c) \( f(h+1) = \) ___________

3. Which of these graphs represent
   a) Relations ? __________________
   b) Functions ? __________________

1) 2) 3) 4)

4. Which of these represent functions ? __________________
   1) \( y = |x| \) 2) \( y^2 = -4x \) 3) \( y = x/4 \) 4) \( 2x - y = 6 \) 5) NONE of these

5. What is the domain of
   a) __________________
   b) __________________
1. Match each graph with the corresponding function. Use the equation that best represents the graph.

\[ y = x^2 + 2x + 1 \quad f(x) = |x + 2| \quad g(x) = 3x - 4 \quad h(x) = 2^x \quad \text{NONE of these} \]

a) _______________  
b) _______________

c) _______________  
d) _______________

2. What is the domain of \( f(x) = \frac{x - 4}{x^2 - 9} \)?

3. Use the function from above. Find \( f(0) \) = __________  
\( f(-3) \) = ______________

4. Sketch the graph of \( y = 2x - 4 \)

5. What is the x-intercept of \( 2x + 3y = 12 \) ?

6. Find the slope of the line that passes through the points \( A(2, 4) \) and \( B(-2, 4) \).