1. \( x + 2 = 1 \) \( \rightarrow \) \( x = \) __________

2. Find the smallest
   a) natural number \( \rightarrow \) __________
   
   b) whole number \( \rightarrow \) __________

3. True or False.
   __________ a) all real numbers are positive or negative.

   __________ b) an equation of the form \( ax + by = 1 \) represents a line.

   __________ c) an equation of the form \( ax^2 + bx + c = y \) represents a parabola.

4. Find the x-intercept of
   a) \( x + 2y = 4 \) \( \rightarrow \) __________

   b) \( y = x^2 - 4 \) \( \rightarrow \) __________

5. Find the y-intercept of
   \( y = x^2 + 2x + 4 \)
1. A __________________________ is a collection of objects having a well-defined property in common.

   The __________________________ is the set that contains all objects under consideration for the given experiment.

   We say that set A is a ______________________ of the set B if for every x in A (x ∈ A), then x is also in B (x ∈ B).

2. Which of these two sets are equal to set A = { 1, 2, 3 } ? → __________________

   { 1, 3, 2 }   { 1, 1, 2, 3, 3 }   both are    neither is

3. True or False
   If A = { a, e, i, o, u }, then
   __________ a) a ∈ A
   __________ b) { a, e } ∈ A
   __________ c) φ ⊂ A
   __________ d) { a, e, i, o, u } ⊂ A

4. IF two sets A and B do not have an object (element in common), we say that A and B are ________________

5. How many subsets does the set C = { x : x is a whole number < 6 } have? ______________

6. A student has five distinct books at home. He decides to go to the library and study — if his backpack is opened, how many different groups of books could it contain? ______________
1. Let \( U = \{ a, e, i, o, u \} \), \( A = \{ a, e, i \} \), \( B = \{ e, i, o \} \), \( C = \) set of all consonants in the English alphabet.

a) Find \( C \).
\( C = \) ___________________

b) Find \( B^\prime \).
\( B^\prime = \) ___________________

c) Find \( A \times B = \) ___________________

d) Find \( n( A \times B ) \).
\( n(A \times B) = \) ___________________

e) Find \( A \cap B \).
\( A \cap B = \) ___________________

2. Complete by filling in the blank.

a) The ________________ of sets \( A \) and \( B \) is the set that contains all objects in the universal set \( U \) that are found in \( A \) or in \( B \) or in both \( A \) and \( B \).

b) The ________________ of \( A \), written \( A^\prime \), is the set that contains all objects in \( U \) that are not in the set \( A \).

3. If \( n(A ) = 20 \), \( n(B ) = 10 \), and \( n( A \cap B ) = 2 \), then use a Venn Diagram to find \( n(A \cup B ) \). ________________

4. Use a Venn Diagram to shade the following sets.

a) \( A \cap B \).

b) \( A \setminus B \)
1. There are four doors to exit a building, three possible modes of transportation (car, bus, trolley), and two ways to enter the final destination.

   How many different paths (ways of leaving and ending up) are possible from beginning to end of destination? _______

2. Find \( n(A \cup B) \) if \( A = \{ x \mid x > 3 \} \) and \( B = \{ x \mid x < 7 \} \) and \( U = \{ x \mid x \text{ is a natural number} < 10 \} \).

   \( n(A \cup B) = \) __________

3. How many different three digit area codes are possible if any digit can be used in any of the three positions?

   \# of different area codes = __________

4. Given \( A = \{ a, b \} \), \( B = \{ c, d, e \} \) find \( n(A \times B \times A) = \) __________

5. A couple is to be selected as a Prom Queen and King. How many distinct couple are possible if there are 20 young men that can be selected as Prom King and 25 young ladies that can be selected as Prom Queen?

   \# of distinct couples = ______________

7. Suppose that a four sided die is rolled. Write a sample space that describes the possible outcomes.

   \( S = \) ______________

8. Find the value of \( x \) if \( \frac{1}{2} + \frac{1}{4} + x = 1 \)

9. We say that a sample space \( S = \{ s1, s2, s3 \} \) has uniform probability if each of the elementary events has the same probability of occurring.

   What is the probability of each elementary event? ______________

10. A class consists of 12 men and 8 ladies. Two of the men wear glasses and 3 of the ladies do also. A student is selected at random. What is the probability that the student is

    a) a male student? ______________

    b) a student that wears glasses? ______________

    c) a male student that wears glasses? __________ 

    d) male if the student is known to wear glasses? _____
1. A graduate class consists of three students. One of these three students is to be chosen at random – but based on their work the second student is twice as likely to be selected as the first and the third student is three times as likely to be selected as the first.

Write a sample space: \( S = \) ____________________________________

Write a probability distribution for \( S \)

Does \( S \) have uniform probability? ________________

2. If \( S \) is some sample space with event \( A \) in \( S \), then

a) \( P( S ) = \) ________________

b) ______ \( \leq P( A ) \leq ______

3. True or False.

______________ a) If \( A \) and \( B \) are disjoint then, then \( P( A \cap B ) = \phi \)

______________ b) If \( E = \{ s1, s2 \} \) and \( S = \{ s1, s2, s3 \} \), then \( P(E) = 2/3 \)

______________ c) Given events \( A \) and \( B \), \( P(A \cup B) = P(A) + P(B) \).

4. A student is to be selected from a class of 12 and paired off with another student also selected at random. What is the probability that the first two students in the alphabet are selected.

5. A single 10-sided die is rolled. The faces are labeled as 1, 1, 2, 2, 2, 3, 3, 3, 3, 4

Write a sample space and the probability distribution of that sample space. Does it have uniform probability.

6. A class consists of 20 students. 12 of them eat breakfast in the morning and 14 eat lunch. 8 of the students eat both meals. Then how many do not eat either meal?
1. Complete the following formulas.
   a) If A and B are mutually exclusive then the simplest way to write
   \( n(A \cup B) \) is ? \( n(A \cup B) = \) _______________________

   b) For any two events A and B \( P(A \cup B) = \) _______________________

   c) Given some event A, \( P(A') = \) _______________________

2. If \( S = \{s_1, s_2, s_3, s_4\} \) with \( P(s_1) = 2/11 \), \( P(s_2) = 3/11 \), \( P(s_3) = 4/11 \) and \( E = \{s_1, s_4\} \), then find
   a) \( P(S) = \) ____________
   b) \( P(E) = \) ____________

3. If E and F are mutually exclusive, then \( P(E \cap F) = \) ______________

4. If \( P(E) = 0.6 \) and \( P(F) = 0.8 \), could E and F be mutually exclusive? Explain.

5. Use the given Venn Diagram to find
   a) \( P(A) = \) ______________
   b) \( P(A \cup B) = \) ______________
   c) \( P(A \cap B') = \) ______________

6. Find the odds in favor of tossing three identical outcomes in a toss of three fair coins.

7. A test is given to see if people enjoy drinking Coke, Pepsi, or Big Red. A sample of 100 students is taken.
   40 liked coke; 45 liked Pepsi; 25 liked Big Red; 7 liked all three
   15 liked both coke and Pepsi; 11 liked coke and Big Red; 13 liked Pepsi and Big Red

   One of these 100 individuals is selected at random. What is the probability that the person selected
   a) does not like any of the three? ______________________
   b) likes only coke? ______________
1. Complete the formulas.
   a) \( P( A \mid B ) = \) ___________________
   b) If \( A \) and \( B \) are independent, then \( P( A \cap B ) = \) ____________________
   c) If \( A \) and \( B \) are any type of events (independent or not), then \( P( A \cap B ) = \) ____________________

2. Given that \( P( A ) = 0.4 \) and \( P(B) = 0.5 \), find
   a) (If \( A \) and \( B \) are known to be mutually exclusive)
      \( P(A \mid B) = \) ____________  \( P(A \cap B) = \) __________
   b) (If \( A \) and \( B \) are known to be mutually exclusive)
      \( P(A \mid B) = \) ____________  \( P(A \cup B) = \) __________

3. A box contains 6 pairs of socks (twelve total socks). None of the pairs are similar to each other (the socks in each pair are identical).
   If you have one favorite pair and you select two socks at random, what is the probability that the socks you select will be your favorite matching pair?
     __________

4. A class consists of 8 freshmen, 5 sophomores, and 3 juniors. Two students are selected at random. What is the probability that the first student selected is a sophomore and the second student is a junior?
     __________

5. A bag contains 3 red marbles, 4 blue marbles, and 3 white ones. You select two marbles.
   a) If this is done without replacement, what is the probability that
      both are red? __________
      the first one is red and the second one is white? __________
      One is red and the other one is white? __________
   b) If this is done with replacement, what is the probability that
      one is red and the second one is white? __________
1. Find

\[ 0! = \quad \quad \quad 290! / 289! = \quad \quad \quad \]

\[ n(n-1)(n-2)(n-3) \cdots 3 \cdot 2 \cdot 1 = \quad \quad \quad \]

2. Which of these best describes the statement “no repetitions and orders matters” → ______________________

- Permutations
- Combinations
- both do
- neither does

3. Use the four choices above to describe the following two

a) Two individuals are to be selected to attend a food expo in San Antonio. There are 8 members available. How many different choices are possible?

→ ______________________

b) Two students are selected at random. The first student to be selected is to receive an automatic A while the second one will receive an F. If the class consists of 15 students, then how many different ways could the two be selected?

→ ______________________

4. A student has three choices of an a morning class, four choices of an afternoon class, and any one of two choices for an evening class. The student plans to take one class from each group. The selection of one does not affect the other choices. How many groups of three classes are possible?

5. A die is rolled four times. What is the probability that (assume a fair six-sided die)

a) all are sixes? ______________

b) none are sixes? ______________

c) at least one is a six? ______________
1. Write down the formula for each of the following
   a) \( C(n, r) = \)  
   b) \( P_r = \)

2. Simplify the following radicals – show work (no calculator – except to check your work)
   a) \( \frac{24!}{4! \cdot 23!} = \)  
   b) \( \frac{1000!}{999!} = \)
   c) \( P(12, 2) = \)  
   d) \( C(200, 198) = \)
   e) \( P(20, 20) = \)  
   f) \( C(30, 0) = \)

3. A group of ten students is to visit the local museum. Once there three will be selected at random to demonstrate an experiment. How many distinct groups are possible? __________
   If there are six girls and four boys, what is the probability that all of them are girls? __________

4. You will try to create a code with all 26 letters of the alphabet and the ten digits. The code will consist of any three of these symbols. The order the symbols are written in should be important. How many distinct symbols are possible?

5. A student will use the letters \{a, b, c, d, e, f\} to create four letter “words”. The letters may be used more than once. How many such letters are possible?

6. Five cards are selected from a standard deck. What is the probability that your hand consists of exactly 2 aces?
1. Four cards are selected at random from a standard deck of cards (52). What is the probability that
   a) all are aces? ________________

   b) none are aces? ______________

   c) at least one is an ace?

2. The probability that a student answers a question on a quiz correctly is 0.6
   The question is asked again at a later time on a test. The probability that the student will answer that question correctly
   on the exam if he answered it correctly on the quiz is 0.9
   The question that the student will answer the question correctly if he answered it wrong on the quiz is 0.8

   What is the probability that
   a) the student will answer the question correctly on the quiz and on the test? ______________

   b) the student will answer it correctly on the exam? ______________

   c) answered the question correctly on the quiz if he is known to have answered the question correctly on the exam?

3. A five problem multiple choice quiz is given with three answers per question. The student guesses at every question.
   What is the probability that the student
   a) will get all questions wrong? ______________

   b) at least one right? ______________

   c) will get the first one right and the next four wrong? ______________
1. A Bernoulli Experiment consists of 2 outcomes we call them a ______ or a ________

2. A four sided die with faces labeled 1, 1, 2, 3 is rolled. Each side is equally likely to occur. The die is rolled five times.
   a) Write a sample space for a single roll. S = ____________________
   b) probability distribution (single roll):
   c) What is the probability that in five rolls
      a) all one’s ? __________________________
      b) at least one of the rolls is a one ? __________
      c) you get exactly one roll being a one ? __________

3. The following is considered a binomial experiment. Each day of the week for 4 weeks (five days/week) you get a coke from the coke machine. The machine has a probability of 0.1 of stealing your money on any given day. What is the probability that you will “lose” your money
   a) every day ? __________
   b) at least one day ? __________
   c) exactly four days ? __________

4. A box contains 8 cokes, 6 sprites, 4 Dr. Peppers, and 2 Pepsies. Four different individuals walk up to the box and grab a drink at random (to drink). What is the probability that
   a) all were cokes ? ________
   b) at least one was a coke ? __________
   c) exactly three were cokes ? __________
1) When are A and B independent events - \( P( A ) = 0.2 \) and \( P( B ) = 0.5 \) - We know that \( P( A | B ) = P( A ) \) and \( P( B | A ) = P(B) \). Give me another reason.

2. Show me an example of a Venn Diagram in which the two events are mutually exclusive. Use numbers (probabilities) to convince me.

3. How many combinations can be taken from 100 objects if you are selecting 100 objects at a time with no repetitions?

4. A binomial experiment can only have two outcomes. True or False.

5. A Bernoulli experiment has two outcomes called a success and a ______________

6. Write down the Binomial Formula - probability of x successes =

7. Hamburger shop makes burgers with buns and meat. There are an additional five choices. If a customer comes and orders a hamburger, how many different types of burgers could be ordered?

8. In the previous problem; the customer has two choice out of four liquid condiments and three out four solid (condiments?)

   How many different burgers are possible?

9. A three digit number is to be created using any one of the ten known single digits. How many numbers are possible if the digit can be repeated?
1. A normal curve with mean 25 and variance 9 has
   a) total area under the curve equal to ___________ units
   b) the inflection points of such a curve occur at ______________ and ___________
2. A standard normal curve has mean ______________ and standard deviation ___________
3. Find the area
   a) to the left of 3 under a standard normal curve ______________
   b) to the right of −2 under a standard normal curve _____________
4. Given data that is normally distributed – mean = 20 and variance = 4, find the area to the right of 27. ______________
5. Write down the binomial formula.
6. A box of 100 pencils is opened. The company that produces the pencils advises you that 2% of all pencils produced by the company will be defective.
   How many pencils do you expect to be defective? ___________
   What is the probability that 4 will be defective? ________
   What is the probability that more than 4 will be defective? (Solve by approximating with a normal curve) __________
7. What does a binomial r.v. measure? ________________________________
8. Find the standard deviation of the following r.v.

<table>
<thead>
<tr>
<th>X = x</th>
<th>P( X = x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1/11</td>
</tr>
<tr>
<td>1</td>
<td>3/11</td>
</tr>
<tr>
<td>2</td>
<td>7/11</td>
</tr>
</tbody>
</table>
9. Draw a histogram of the r.v. from #8.

10. A six-sided die is rolled. The sides are labeled as 1, 1, 1, 2, 3, 3. Let the r.v. X represent the value of the outcome. Write down all the possible values of X.

What is the expected value of X? __________

Variance? __________

11. A game is played as follows;

an ace is drawn – you win $20 plus you get your $2 back

a face card is drawn – you get your $2 but you do not win anything else

any other card – you lose your $2.

From the dealers viewpoint – what is the expected value? ________________

If you as a player played this game 100 times, then how much would you expect to wind up winning (or losing)? _____

12. An insurance company currently sells $100,000 at a cost of $1000 per year. It is known that there is a 0.1% chance that a person in this category will die within the year. What is the expected value of this policy?
1. Which one (1) one these equations does not represent a line? Circle your answer.

\[ y = 1 - 3x \quad y = x^2 + 2x - 1 \quad x = 3 \quad y = -4 \]

If it does not represent a line, what does it represent? ________________

2. What is the domain of

a) \( f(x) = \frac{3}{x+2} \)  
   Domain: ________________________________

b) \[ \text{Diagram} \]  
   Domain: ________________________________

3. What is the range of

a) \( y = x^2 + 2 \)  
   graph:

   \[ \text{Graph} \]  
   Range: ________________________________

b) \[ \text{Diagram} \]  

4. Which of these is (are) not functions

a) \#2b \( \rightarrow \) relation, function, or both

b) \#3b \( \rightarrow \) relation, function, or both

c) \( y^2 = 4x \) \( \rightarrow \) relation, function, or both
1. Given $f(x) = \frac{x - 2}{3}$, $g(x) = 2$, $h(x) = 1 - x^2$, $t(x) = x$

Find
a) $f(2) = \underline{\hspace{2cm}}$  
   $g(h) = \underline{\hspace{2cm}}$

b) $h(-1) = \underline{\hspace{2cm}}$  
   $t(v) = \underline{\hspace{2cm}}$

e) $f(h + 3) = \underline{\hspace{2cm}}$

2. Given a function $f(x) = x^2 + 2$ find
the point on the curve represented by $f(x)$ at $x = -1$ and the point at $x = 2$.  

   What is the slope of the line that connects the two points above?  
   This is called the average rate of change of $f(x)$ as $x$ changes from -1 to 2.

3. Find each of the following limits.
   a) $\lim_{x \to 3} \frac{-2}{x} = \underline{\hspace{2cm}}$
   b) $\lim_{x \to 2} (1 - x) = \underline{\hspace{2cm}}$

   c) $\lim_{x \to 3} \frac{x}{x - 3} = \underline{\hspace{2cm}}$
   d) $\lim_{x \to -3} \frac{x + 3}{x^2 - 9} = \underline{\hspace{2cm}}$

   e) $\lim_{h \to 0} \frac{x + h + 2}{h} = \underline{\hspace{2cm}}$

4. What is the slope of the line that passes through the points (-2, -3) and (4, -2)?  
   What is the slope of a horizontal line?  

5. If $f(x) = 3x - 2$, find $\frac{f(2 + h) - f(2)}{h} = \underline{\hspace{2cm}}$
1. Given

\[ f(x) = \frac{x}{x - 2} \quad \text{and} \quad g(x) = \frac{x + 2}{x^2 - 4} \]

Find

a) \( f(2) = \) _____________________  
b) \( g(2) = \) _______________

c) \( \lim_{x \to 2} f(x) = \) ________  
d) \( \lim_{x \to 2} g(x) = \)

e) \( \lim_{x \to \infty} f(x) = \) ________  
f) \( \lim_{x \to \infty} g(x) = \)

g) point(s) of discontinuity of \( f(x) \)  
h) points of discontinuity of \( g(x) \)

2. Use the \( x \) and \( y \)-intercepts to sketch the graph of

\[ f(x) = 2 - 3x \]

3. Use the fact that \( g(x) = 1 - 2x - x^2 \) represents a parabola to sketch a graph by first finding the vertex.
Quiz #1 - Answers

1. A ____________________________ is a collection of objects having a well-defined property in common.
   
   set

   The __________________________ is the set that contains all objects under consideration for the given experiment.

   universal set

   We say that set A is a ______________________ of the set B if for every x in A (\( x \in A \)), then x is also in B (\( x \in B \)).

   subset

2. Which of these two sets are equal to set \( A = \{ 1, 2, 3 \} \)? → ________________ both are

   \{ 1, 3, 2 \}   \{ 1, 1, 2, 3, 3 \}   both are    neither is

3. True or False
   
   If \( A = \{ a, e, i, o, u \} \), then

   _________ a) \( a \in A \) → true

   _________ b) \( \{ a, e \} \in A \) → false

   _________ c) \( \phi \subset A \) → true

   _________ d) \( \{ a, e, i, o, u \} \subset A \) → true

4. IF two sets A and B do not have an object (element in common), we say that A and B are ________________

   disjoint

5. How many subsets does the set \( C = \{ x : x \text{ is a whole number } < 6 \} \) have?

   \( C = \{ 0, 1, 2, 3, 4, 5 \} \) → C has \( 2^6 \) subsets = 64

6. A student has five distinct books at home. He decides to go to the library and study – if his backpack is opened, how many different groups of books could it contain?

   \[ 2^5 = 32 \]
Quiz #2 – Answers

1. Let $U = \{a, e, i, o, u\}$, $A = \{a, e, i\}$, $B = \{e, i, o\}$, $C =$ set of all consonants in the English alphabet.

a) Find set $C$. $C = \emptyset$

b) Find $B'$. $B' = \{a, u\}$

c) Find $A \times B = \{(a,e), (a,i), (a,o), (e,e), (e,i), (e,o), (i,e), (i,i), (i,o)\}$

d) Find $n(A \times B)$.

$$n(A \times B) = 3 \times 3 = 9$$

e) Find $A \cap B$. $A \cap B = \{e, i\}$

2. Complete by filling in the blank.

a) The __________________ of sets $A$ and $B$ is the set that contains all objects in the universal set $U$ that are found in $A$ or in $B$ or in both $A$ and $B$. \[\text{union}\]

b) The __________________ of $A$, written $A'$, is the set that contains all objects in $U$ that are not in the set $A$. \[\text{complement}\]

3. If $n(A) = 20$, $n(B) = 10$, and $n(A \cap B) = 2$, then use a Venn Diagram to find $n(A \cup B)$.

Draw the Venn Diagram:

since $n(A \cap B) = 2$, then $A - B = 18$ and $B - A = 8$, so $n(A \cup B) = 28$

4. Use a Venn Diagram to shade the following sets.

a) $A \cap B$. \[\text{Draw a Venn Diagram and shade where A and B intersect.}\]

b) $A - B$. \[\text{Draw a Venn Diagram and shade the portion of set A that excludes any part of set B.}\]
1. There are four doors to exit a building, three possible modes of transportation (car, bus, trolley), and two ways to enter the final destination.

How many different paths (ways of leaving and ending up) are possible from beginning to end of destination? ________

\[ \text{answer: } 4 \cdot 3 \cdot 2 = 24 \]

2. Find \( n ( A \cup B ) \) if \( A = \{ x \mid x > 3 \} \) and \( B = \{ x \mid x < 7 \} \) and \( U = \{ x \mid x \text{ is a natural number} < 10 \} \).

\[ n ( A \cup B ) = \] ________  \hspace{1cm} \text{answer: } A \cup B = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9 \} \rightarrow n(A \cup B) = 9

3. How many different three digit area codes are possible if any digit can be used in any of the three positions?

\[ \# \text{of different area codes } = \] ________  \hspace{1cm} \text{answer: } 10 \cdot 10 \cdot 10 = 1000

4. Given \( A = \{ a, b \} \), \( B = \{ c, d, e \} \) find \( n ( A \times B \times A ) = \) ________  \hspace{1cm} \text{answer: } n( A \times B \times A ) = 2 \cdot 3 \cdot 2 = 12

5. A couple is to be selected as a Prom Queen and King. How many distinct couple are possible if there are 20 young men that can be selected as Prom King and 25 young ladies that can be selected as Prom Queen?

\[ \# \text{of distinct couples } = \] ________  \hspace{1cm} \text{answer: } 20 \cdot 25 = 500

6. Suppose that a four sided die is rolled. Write a sample space that describes the possible outcomes.

\[ S = \] ________  \hspace{1cm} \text{answer: } S = \{ 1, 2, 3, 4 \}

8. Find the value of \( x \) if \( \frac{1}{2} + \frac{1}{4} + x = 1 \) ________ \hspace{1cm} \text{answer: } x = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}

9. We say that a sample space \( S = \{ s1, s2, s3 \} \) has uniform probability if each of the elementary events has the same probability of occurring.

What is the probability of each elementary event? ________  \hspace{1cm} \text{answer: } 1/3

10. A class consists of 12 men and 8 ladies. Two of the men wear glasses and 3 of the ladies do also. A student is selected at random. What is the probability that the student is

a) a male student? ________  \hspace{1cm} \text{answer: } 12/20

b) a student that wears glasses? ________  \hspace{1cm} \text{answer: } 5/20

c) a male student that wears glasses? ________

d) male if the student is known to wear glasses? ________  \hspace{1cm} \text{answer: } 2/5
1. A graduate class consists of three students. One of these three students is to be chosen at random – but based on their work the second student is twice as likely to be selected as the first and the third student is three times as likely to be selected as the first.

Write a sample space: \[ S = \{ s_1, s_2, s_3 \} \]

Write a probability distribution for \( S \)

\[ \text{prob. model: } P(s_1) = x, \ P(s_2) = 2x, \ P(s_3) = 3x \] \[ \rightarrow x + 2x + 3x = 1 \]
\[ 6x = 1 \rightarrow x = 1/6 \]

\[ P(s_1) = 1/6, \ P(s_2) = 2/6, \ P(s_3) = 3/6 \]

Does \( S \) have uniform probability? \underline{NO!}

2. If \( S \) is some sample space with event \( A \) in \( S \), then

\( a) \ P( S ) = \underline{1} \)
\( b) \underline{0} \leq P( A ) \leq 1 \)

3. True or False.

\( a) \text{ If } A \text{ and } B \text{ are disjoint then, then } P( A \cap B ) = \phi \) \underline{answer: false}

\( b) \text{ If } E = \{ s_1, s_2 \} \text{ and } S = \{ s_1, s_2, s_3 \}, \text{ then } P(E) = 2/3 \) \underline{answer: false}

\( c) \text{ Given events } A \text{ and } B, \ P(A \cup B) = P(A) + P(B). \) \underline{answer: false}

4. A student is to be selected from a class of 12 and paired off with another student also selected at random. What is the probability that the first two students in the alphabet are selected.

\underline{OMIT: ask me in class}

5. A single 10-sided die is rolled. The faces are labeled as 1, 1, 2, 2, 2, 3, 3, 3, 3, 4

Write a sample space and the probability distribution of that sample space. Does it have uniform probability.

\underline{Answer: } \[ S = \{ 1, 2, 3, 4 \} \rightarrow P(1) = 2/10, \ P(2) = 3/10, \ P(3) = 4/10, \ P(4) = 1/10 \]

6. A class consists of 20 students. 12 of them eat breakfast in the morning and 14 eat lunch. 8 of the students eat both meals. Then how many do not eat either meal?

\underline{answer: 2}
1. Complete the following formulas.
   a) If A and B are mutually exclusive then the simplest way to write
      \( n(A \cup B) \) is? \( n(A \cup B) = \) ____________________________________
      \text{answer: } n(A \cup B) = n(A) + n(B)
   
   b) For any two events A and B \( P(A \cup B) = \) ______________________
      \text{answer: } P(A \cup B) = P(A) + P(B) - P(A \cap B)
   
   c) Given some event A, \( P(A') = \) ______________________
      \text{answer: } P(A') = 1 - P(A)

2. If \( S = \{ s1, s2, s3, s4 \} \) with \( P(s1) = 2/11, \ P(s2) = 3/11, \ P(s3) = 4/11 \) and \( E = \{ s1, s4 \} \), then find
   a) \( P(S) = \) ____________ \text{answer: } P(S) = 1
   b) \( P(E) = \) ____________ \text{answer: } 4/11

3. If E and F are mutually exclusive, then \( P(E \cap F) = \) ______________
   \text{answer: } P(E \cap F) = 0

4. If \( P(E) = 0.6 \) and \( P(F) = 0.8 \), could E and F be mutually exclusive? Explain.
   \textbf{No! } P(E \cup F) > 1.

5. Use the given Venn Diagram to find
   a) \( P(A) = \) ______________
   \text{answer: } 0.7
   
   b) \( P(A \cup B) = \) ______________
   \text{answer: } 0.9
   
   c) \( P(A \cap B)' = \) ______________
   \text{answer: } 0.6

6. Find the odds in favor of tossing three identical outcomes in a toss of three fair coins.
   \text{answer: } 2 : 6

7. A test is given to see if people enjoy drinking Coke, Pepsi, or Big Red. A sample of 100 students is taken.

   40 liked coke; 45 liked pepsi; 25 liked big red; 7 liked all three
   15 liked both coke and pepsi; 11 liked coke and big red; 13 liked pepsi and big red

   One of these 100 individuals is selected at random. What is the probability that the person selected
   a) does not like any of the three? ______________
   \text{answer: } 22/100
   
   b) likes only coke? ______________
   \text{answer: } 21/100
1. Complete the formulas.
   a) $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

   **Answers:** $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$; $P(A \cap B) = P(A) \cdot P(B)$ if they are independent

   b) If $A$ and $B$ are independent, then $P(A \cap B) = \frac{P(A)}{P(B)}$

   **Answer:** $P(A \cap B) = P(A) \cdot P(B)$ --- always.

2. Given that $P(A) = 0.4$ and $P(B) = 0.5$, find
   a) (If $A$ and $B$ are known to be independent)
      $P(A \cap B) = \frac{P(A)}{P(B)}$

      **Answer:** $P(A \cap B) = 0.4$; $P(A) \cdot P(B) = (0.4) \cdot (0.5) = 0.2$

   b) (If $A$ and $B$ are known to be mutually exclusive)
      $P(A \cup B) = P(A) + P(B)$

      **Answer:** $P(A \cup B) = 0.4 + 0.5 = 0.9$

3. A box contains 6 pairs of socks (twelve total socks). None of the pairs are similar to each other (the socks in each pair are identical)

   If you have one favorite pair and you select two socks at random, what is the probability that the socks you select will be your favorite matching pair?

   **Answer:** $\frac{2}{12} \cdot \frac{1}{11} = \frac{1}{66}$

4. A class consists of 8 freshmen, 5 sophomores, and 3 junior. Two students are selected at random. What is the probability that the first students selected is a sophomore and the second student is a junior?

   **Answer:** $\frac{5}{16} \cdot \frac{3}{16}$

5. A bag contains 3 red marbles, 3 blue marbles, and 4 white ones. You select two marbles.
   a) If this is done without replacement, what is the probability that both are red?

      **Answer:** $\frac{3}{10} \cdot \frac{2}{9} = \frac{1}{15}$

   b) If this is done with replacement, what is the probability that one is red and the other one is white?

      **Answer:** $\frac{3}{10} \cdot \frac{4}{10} + \frac{4}{10} \cdot \frac{3}{10} = \frac{24}{100}$
1. Find

\[ 0! = \] __________  \[ \quad 290! / 289! = \] __________

answer: 1  \quad answer: \[ \frac{290 \cdot 289!}{289!} = 290 \]

\[ n(n-1)(n-2)(n-3) \cdots (3)(2)(1) = \] _______________

answer: \[ n! \]

2. Which of these best describes the statement “no repetitions and orders matters” → ________________

**Permutations**  \quad **Combinations** \quad both do \quad neither does

3. Use the four choices above to describe the following two

a) Two individuals are to be selected to attend an food expo in San Antonio. There are 8 members available. How many different choices are possible ?

→ ________________

**answer:** Combination

b) Two students are selected at random. The first student to be selected is to receive an automatic A while the second one will receive an F. If the class consists of 15 students , then how many different ways could the two be selected ?

→ ________________  answer: Permutation

4. A student has three choices of an a morning class, four choices of an afternoon class, and any one of two choices for an evening class. The student plans to take one class from each group. The selection of one does not affect the other choices. How many groups of three classes are possible ?

answer: cross product \[ 4 \cdot 3 \cdot 2 = 24 \]

5. A die is rolled four times. What is the probability that ( assume a fair six-sided die )

a) all are sixes ? ________________  \quad b) none are sixes ? ________________

answer: \[ (1/6)^4 \]  \quad answer: \[ (5/6)^4 \]

c) at least one is a six ? ________________ answer: \[ 1 - (5/6)^4 \]
1. Write down the formula for each of the following
   a) \( C(n, r) = \)  
   b) \( nP_r = \frac{n!}{(n-r)!} \)

2. Simplify the following radicals – show work (no calculator – except to check your work)
   a) \( \frac{24!}{4! \cdot 23!} = \)  
   b) \( \frac{1000!}{999!} = \)  

   answer: \( \frac{24 \cdot 23!}{4(3)(2)(1) \cdot 23!} = \frac{24}{4(3)(2)(1)} = 1 \)  
   answer: \( \frac{1000 \cdot 999!}{999!} = \frac{1000}{2} = \frac{199000}{2} = 99500 \)

   c) \( P(12, 2) = \)  
   d) \( C(200, 198) = \)  

   answer: \( 12 \cdot 11 = 132 \)  
   answer: \( \frac{200 \cdot 199}{2} = 19900 \)

   e) \( P(20, 20) = \)  
   f) \( C(30, 0) = \)  

   answer: \( 20! \)  
   answer: \( 1 \)

3. A group of ten students is to visit the local museum. Once there three will be selected at random to demonstrate an experiment. How many distinct groups are possible? \( C(10, 3) = 10 \cdot 9 \cdot 8 / 6 = 120 \)

If there are six girls and four boys, what is the probability that all of them are girls? \( \frac{C(6, 3) \cdot C(4, 0)}{C(10, 3)} = \frac{6(5)(4)}{3(2)(1)} \cdot \frac{10(9)(8)}{3(2)(1)} = \frac{6(5)(4)}{10(9)(8)} = \frac{1}{6} \)

4. You will try to create a code with all 26 letters of the alphabet and the ten digits. The code will consist of any three of these symbols. The order the symbols are written in should be important. How many distinct symbols are possible? (with repetitions) \( 36 \cdot 36 \cdot 36 = \)  

5. A student will use the letters \{a, b, c, d, e, f\} to create four letter “words”. The letters may be used more than once. How many such letters are possible? \( C(6, 4) = 6(5)/2 = 15 \)

6. Five cards are selected from a standard deck. What is the probability that your hand consists of exactly 2 aces? \( C(4, 2) \cdot C(48, 3) / C(52, 5) \).
1. Four cards are selected at random from a standard deck of cards (52). What is the probability that
   a) all are aces? \( \frac{\binom{4}{4}}{\binom{52}{4}} \)
   b) none are aces? \( \frac{\binom{48}{4}}{\binom{52}{4}} \)
   c) at least one is an ace? \( 1 - \frac{\binom{48}{4}}{\binom{52}{4}} \)

2. The probability that a student answers a question on a quiz correctly is 0.6
   The question is asked again at a later time on a test. The probability that the student will answer that question correctly
   on the exam if he answered it correctly on the quiz is 0.9
   The question that the student will answer the question correctly if he answered it wrong on the quiz is 0.8
   What is the probability that
   a) the student will answer the question correctly on the quiz and on the test? \( 0.54 \)
   b) the student will answer it correctly on the exam? \( 0.54 + 0.32 = 0.86 \)
   c) answered the question correctly on the quiz if he is known to have answered the question correctly on the exam?

\[ \frac{0.54}{0.86} = \frac{54}{86} \]

3. A five problem multiple choice quiz is given with three answers per question. The student guesses at every question.
   What is the probability that the student
   a) will get all questions wrong? \( \frac{2}{3} \)^5 \( = \frac{32}{243} \)
   b) at least one right? \( 1 - \frac{2}{3} \)^5 \( = 1 - \frac{32}{243} \)
   c) will get the first one right and the next four wrong? \( \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{16}{243} \)
1. A Bernoulli Experiment consists of 2 outcomes we call them a _______________ or a _______________
   success and a failure

2. A four sided die with faces labeled 1, 1, 2, 3 is rolled. Each side is equally likely to occur. The die is rolled five times.
   a) Write a sample space for a single roll. S = ________________________________
      S = { 1, 2, 3 }
   b) probability distribution(single roll):
      P( 1 ) = 2/4 = ½ , P( 2 ) = ¼ , P ( 3 ) = 1/ 4
   c) What is the probability that in five rolls
      1) all one’s ? ___________________________
      C( 5, 5 ) ( ½) 0 = 1/ 32
      b) at least one of the rolls is a one ? ________________
      1 - C( 5, 0 ) ( ½) 5 = 31/32
      c) you get exactly one roll being a one ? ________________
      C(5, 1) ( ½) 1 ( ½) 4 = 5/32

3. The following is considered a binomial experiment. Each day of the week for 4 weeks (five days / week) you get a coke from the coke machine. The machine has a probability of 0.1 of stealing your money on any given day. What is the probability that you will “lose” your money
   a) every day ? _____________
      C(20, 20 ) ( 0.1)20 ( 0.9)0 =
   b) at least one day ? ____________
      1 - C( 20, 0 ) (0.1)0(0.9)20 =
   c) exactly four days ? ____________
      C(20, 4)(0.1)4(0.9)16

4. A box contains 8 cokes, 6 sprites, 4 Dr. Peppers, and 2 Pepsies. Four different individuals walk up to the box and grab a drink at random(to drink). What is the probability that
   a) all were cokes ? _____________
      C(8,0) / C(20, 4)
   b) at least one was a coke ? ____________
      1 - C(8,0)C(12,4) / C(20,4)
   c) exactly three were cokes ? ____________
      C(8,3)C(12,1) / C(20,4)
1. A normal curve with mean 25 and variance 9 has
   a) total area under the curve equal to ___________ units  **answer: 1 square unit**
   b) the inflection points of such a curve occur at ___________ and __________ at: 22 and 28

2. A standard normal curve has mean ________ and standard deviation ______________  mean of 0 and st. dev. = 1

3. Find the area
   a) to the left of 3 under a standard normal curve ______________
      **answer: since z = 3.00 look in the table and find Area = 0.4987 → but need to add 0.5000 → answer: 0.9987**
   b) to the right of –2 under a standard normal curve ____________
      **answer: z = -2.00 → look in table and find area to the mean = 0.4772 → need to add 0.5000 → answer: 0.9772**
      (had the question been: the area to the left → 0.5000 – 0.4772 = 0.0228 would have been your answer)

4. Given data that is normally distributed – mean = 20 and variance = 4, find the area to the right of 27. ______________
   \[ z = (x - \mu) / \sigma = \frac{27 - 20}{2} = 3.50 \rightarrow \]
   the table does not reach such a value → area to the mean is close to 0.5000
   area to the right of 27 → close to 0.0000

5. Write down the binomial formula. \[ ^nC_r (p)^r (q)^{n-r} \]

6. A box of 100 pencils is opened. The company that produces the pencils advises you that 2 % of all pencils produced by the company will be defective.
   How many pencils do you expect to be defective? ________ **answer: Binomial \rightarrow E(x) = np = 100 (0.02) = 2**
   What is the probability that 4 will be defective? ________ **answer: \( C(100, 4) (0.02)^4 (0.98)^{96} \)**
   What is the probability that more than 4 will be defective? (Solve by approximating with a normal curve) ________
   **answer: more than 4 means four will not be included → find the area greater than 4.5 → \[ z = (4.5 - 2) / \sigma, \text{ where } \sigma = 1.4 \rightarrow \text{look up the z value in table and subtract result from 0.5.} \]**

7. What does a binomial r.v. measure? ________________________________
   the number of successes
8. Find the standard deviation of the following r.v.

\[
\begin{array}{ccc}
X = x & P(X = x) \\
-2 & 1/11 & -2/11 \\
1 & 3/11 & 3/11 \\
2 & 7/11 & 28/11 \\
\end{array}
\]

\[
\text{standard deviation } = \sqrt{35/11 - (15/11)^2}
\]

9. Draw a histogram of the r.v. from #8.

10. A six-sided die is rolled. The sides are labeled as 1, 1, 1, 2, 3, 3. Let the r.v. X represent the value of the outcome.

\[
X = 1, 2, 3
\]

What is the expected value of X? 1 (3/6) + 2 (1/6) + 3(2/6) = 11/6

Variance? 

\[
\begin{array}{ccc}
1 & 3/6 & 3/6 \\
2 & 1/6 & 2/6 \\
3 & 2/6 & 6/6 \\
\end{array}
\]

E(x) = 11/6

\[
\text{Variance } = 25/6 - (11/6)^2
\]

11. A game is played as follows:

- an ace is drawn – you win $20 plus you get your $2 back
- a face card is drawn – you get your $2 but you do not win anything else
- any other card – you lose your $2.

From the dealers viewpoint – what is the expected value? 

\[
-20 \left( \frac{1}{13} \right) + 0 \left( \frac{3}{13} \right) + 2 \left( \frac{9}{13} \right) = -2/13 \rightarrow \text{dealer loses about } $0.15 \text{ to } $0.16 \text{ per game}
\]

If you as a player played this game 100 times, then how much would you expect to wind up winning (or losing)?

You would expect to lose on the average 100 (0.15) = about $15.

12. An insurance company currently sells $100,000 at a cost of $1000 per year. It is known that there is a 0.1% chance that a person in this category will die within the year. What is the expected value of this policy?

\[
E(X) = -0.001 \left( 100000 - 1000 \right) + 0.999(1000) = __________
\]
1. Which one (1) one these equations does not represent a line? Circle your answer.

\[ y = 1 - 3x \quad y = x^2 + 2x - 1 \quad x = 3 \quad y = -4 \]

If it does not represent a line, what does it represent? **A parabola – quadratic function**

2. What is the domain of

a) \[ f(x) = \frac{3}{x + 2} \] Domain: __________________

   *the set of all real numbers except \( x = -2 \)*

b) Domain: __________________ \( \{1, 2, 3\} \)

3. What is the range of

a) \( y = x^2 + 2 \) graph:

   Range: __________________

   *all real number \( y, y \geq 0 \)*

b) Range: \( \{1, 2, 3\} \)

4. Which of these is (are) not functions

a) \#2b → relation, function, or both → **Function and a relation, both**

b) \#3b → relation, function, or both → **only a relation**

c) \( y^2 = 4x \) → relation, function, or both → **only a relation**
1. Given \( f(x) = \frac{x - 2}{3} \), \( g(x) = 2 \), \( h(x) = 1 - x^2 \), \( t(x) = x \)

Find
a) \( f(2) \) . \( \frac{0}{3} \) answer: 0

b) \( g(h) = \frac{2}{3} \) answer: 2

c) \( h(-1) = \frac{1}{3} \) answer: 0

d) \( t(v) = v \) answer: v

e) \( f(h + 3) = \frac{(h + 1)}{3} \) answer: \( \frac{(h + 1)}{3} \)

2. Given a function \( f(x) = x^2 + 2 \) find

the point on the curve represented by \( f(x) \) at \( x = -1 \) and the point at \( x = 2 \).

answer: if \( x = -1 \), then \( y = 3 \) --- also; if \( x = 2 \), then \( y = 6 \) → the two points: \((-1, 3) \) and \((2, 6) \) →

What is the slope of the line that connects the two points above? \( \frac{3}{3} = 1 \) answer: \( m = \frac{3}{3} = 1 \)

This is called the average rate of change of \( f(x) \) as \( x \) changes from -1 to 2.

3. Find each of the following limits.

a) \( \lim_{x \to 3} -2 = -2 \) answer: -2

b) \( \lim_{x \to 2} (1 - x) = -1 \) answer: -1

c) \( \lim_{x \to 3} \frac{x}{x - 3} = \) undefined answer: undefined

d) \( \lim_{x \to -3} \frac{x + 3}{x^2 - 9} = \frac{1}{x - 3} \) answer: \( \frac{1}{x - 3} \)

e) \( \lim_{h \to 0} x + h + 2 \) answer: \( x + 2 \)

4. What is the slope of the line that passes through the points \((-2, -3) \) and \((4, -2) \)? \( \frac{-3 - (-2)}{-2 - 4} = \frac{-1}{-6} = \frac{1}{6} \) answer: \( m = \frac{-3 - (-2)}{-2 - 4} = \frac{-1}{-6} = \frac{1}{6} \)

What is the slope of a horizontal line? \( m = 0 \) answer: \( m = 0 \)

5. If \( f(x) = 3x - 2 \), find \( \frac{f(2 + h) - f(2)}{h} = \frac{3h}{h} = 3 \) answer: \( \frac{3(2 + h) - 2 - (6 - 2)}{h} = \frac{3h}{h} = 3 \)
1. Given

\[ f(x) = \frac{x}{x-2} \quad \text{and} \quad g(x) = \frac{x+2}{x^2-4} \]

Find

a) \( f(2) = \) _____________________  

b) \( g(2) = \) _______________

c) \( \lim_{x \to 2} f(x) = \) __________  

d) \( \lim_{x \to 2} g(x) = \) __________

e) \( \lim_{x \to \infty} f(x) = \) __________  

f) \( \lim_{x \to \infty} g(x) = \) __________

g) point(s) of discontinuity of \( f(x) \)  

h) points of discontinuity of \( g(x) \)

→ ____________________  
→ ____________________

2. Use the x and y-intercepts to sketch the graph of

\( f(x) = 2 - 3x \)

3. Use the fact that \( g(x) = 1 - 2x - x^2 \) represents a parabola to sketch a graph by first finding the vertex.