Name __________________________  Math 1312.030 – Exam I – September 29, 2004

1. Complete the following blanks. Use the actual word not the symbol, for example write complement of A, not A’.
   a) The ________________ of A and B is the set that contains all objects in U that are classified as being in the set A and also in the set B – at the same time.

   b) Two events A and B are said to be ________________ provided  A ∩ B = φ.
      (notice: there are two blanks- fill them in properly)

   c) Events A and B are said to be ________________ events of P( A | B ) = P( A ) or if P( B | A ) = P( B ).

   d) If A and B are independent events, then P( A ∩ B ) = ________________ (simplest form).

2. Complete the following formulas as defined in class – in simplest form
   (reduce a formula to its simplest form with no unnecessary terms).
   a) If E and F are mutually exclusive events, then P( E ∪ F ) = ________________

   b) Given events E and F, P( E | F ) = ________________

3. Shade the region in the Venn Diagram that corresponds to the indicated sets.
   a) A - B = A ∩ B /
   b) ( A ∪ B ) ∩ C /

4. IF A and B are mutually exclusive with P( A ) = 0.4 and P( B ) = 0.2, then find P( A ∩ B ). ____________
5. Let $U$ = set of all letters in the English alphabet. $A$ = set of all vowels, $B = \{a, b, c\}$, $C = \{b, d\}$.
Find (remember that sets always have $\{\}$ unless they are represented by a letter; $C$ or $\{b, d\}$

a) $B \cup C =$ __________________________

b) $A^\prime =$ __________________________

c) $n( A \times B \times C ) =$ __________________________

6. True or False.
Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, and $C = \{4, 5\}$.

_________ a) $A \subset B$ ________ b) $A \times B = 9$

_________ c) $3 \subset A$ ________ d) $5 \in B$

_________ e) $\phi \subset C$ ________ e) $A$ and $C$ are disjoint

7. A card is drawn from a standard deck. What is the probability that the card selected is

a) not an ace? ________________

b) either an ace or a black card? ________________

8. A fair six-sided die is rolled three times and the sequence of outcomes is recorded.

a) How many different sequences are possible? ________________

c) How many sequences in which the first number is a three? ________________

d) What is the probability that the sequence recorded consists of all sixes? ________________

9. If $A$ and $B$ are independent with $P( A ) = 0.4$ and $P( B ) = 0.5$, then find $P( A \cup B )$. ________________
10. **A box contains 8 red balls, 4 blue balls, and 3 yellow balls.** Two balls are drawn and thrown up in the air. After the second one is thrown, you can see both of them in the air. What is the probability that

a) both of them are blue? _____________________

b) the first is blue and the second one is yellow? __________

11. **A box of pencils is labeled as defective** if more than one of the mechanical pencils is defective. If a box contains five pencils, then what is the probability that the box is not defective. Assume that the probability that each individual pencil is defective is 0.1

12. Complete the following Venn Diagram and use it to answer the questions that follow

\[
\begin{align*}
P( A - B ) &= 0.2 \\
P( A ) &= 0.5 \\
P( A \cup B )' &= 0.2
\end{align*}
\]

Find

a) \( P( A \cap B ) = \) ______________

b) \( P( A \mid B ) = \) ______________

c) \( P( B - A ) = \) __________

d) \( P( B ) = \) ______________

d) Are A and B independent? __________ Why or Why not? __________

13. **200 cars enter an intersection, fifty turn left and 70 turn right.** If a car enters an intersection, then what is the probability that the car will turn?
14. Recent studies have shown that out of 100 individuals that sign up for a class
    40 will pass exam 1 and 80 will miss at least one day prior to the first exam.
If there are 92 students that will do either of these two actions (pass exam 1 or miss a day), then how many did
a) neither? __________ 
   b) both? _________

15. Out of 45 students in the classroom we found that
    30 studied for exam 1 20 had a good night sleep 15 ate a good breakfast
    12 studied and slept “good” 8 slept “good” and ate well
    10 studied and ate a good bkfast 4 did all of three of the above 
   a) How many students did neither? ____________________
   b) If a student is selected at random, then what is the probability that the student
      ate well, slept well, but did not study for the exam? _________
   c) What is the probability that the student slept well if he is known to have studied for the exam? ____________

16. A die is rolled twice and the sum of the results are added. What are the odds in favor of the sum being greater
    than 10? 
    __________
17. **During the month of February (28 days) there were 5 days in which rain fell.**
   If a day is selected at random, what are the odds against selecting a day in which rain fell?
   
   _______________

18. **A student takes a four problem multiple choice quiz.** There are five choices per question but he can always eliminate one of the wrong answers. If he guesses at all of the questions from the remaining answers, what is the probability that
   
   a) all of the questions are answered wrong?
   
   _______________

   b) At least one of the questions is answered correctly? (this means: one right or two right or three or four right)
   
   _______

19. **A teacher has a small class consisting of 6 students. He will select students to give a talk.** A student will be selected if the student feels prepared to give a talk. How many different groups could be selected to give a talk?
   
   _______________

20. **A game can only be won by one person** - The probability that Bob wins this game is 0.1, the probability that Bill wins this game is 0.2. What is the probability that neither Bob nor Bill will win?
   
   _______________

21. **A game is played as follows:**
   Five keys are provided – one of the keys will open the room you are in while the other four will not. It takes 2 minutes to test each key. After you leave the room – you must select one of three telephone numbers – dial the right one and you will find the clue. If you dial the wrong one, you have try the next number – it takes one minute to dial each number. Once you find the clue you will have four chances to find the answer to a question. It takes three minutes to look up each an answer. If you are right you are done – otherwise, try the next answer.

   What is the most amount of time that it will take for you to finish the game?
   _______________

   The least amount of time?
   _______________

   What is the probability that you will finish the game in the least amount of time – assuming all of your selections are done at random?
   _______________
22. **There are 30 days in this month.** If I told you that my birthday falls on this month and I gave one chance to guess my bday, then what is the probability that you would guess it correctly? 

23. **A 10-sided die is rolled once. The faces are labeled:** 1, 1, 1, 2, 2, 3, 4, 5, 5, 5

   a) Write a sample space: (remember that a sample space describes what your eyes see as possible outcomes)

   b) Write a probability distribution for your sample space

   c) Does your sample space have uniform probability? Why or Why not?

24. **A sample space consist of 3 different sample points.** The second outcome is three times as likely to occur as the first while the third outcome is four times as likely to occur as the first one.

   a) Write a sample space for this experiment. S = _______________________________________

   b) Write a probability distribution for your sample space.

25. **200 people attend a lecture.** Five tickets are selected to win a prize. Once on stage – one of these individuals is selected at random to selected a marble from a bag that contains 499 white marbles and one black marble. If the black marble is selected, then the person will win $500,000. If you happen to be part of this audience, then what is the probability that you will be selected and end up winning the money?

26. **A class of 50 students consists of 20 female students.** Ten of the male students and four of the female students smoke. A student is selected at random. What is the probability that the student

   a) is male and smokes? ___________  
   b) is male if he is known to smoke?