Discrete Mathematics

Fibonacci and Arithmetic Sequences

1. Fibonacci Sequences
   (a) List the first 12 terms of the Fibonacci sequence (0 through 11).
   (b) Use the formula
       \[ F_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1 + \sqrt{5}}{2} \right)^n - \left( \frac{1 - \sqrt{5}}{2} \right)^n \right] \]
       to find \( F_3 \) and \( F_5 \).
   (c) Use the simplified formula
       \[ F_n \approx \frac{1}{\sqrt{5}} \left( \frac{1 + \sqrt{5}}{2} \right)^n \]
       to find \( F_6 \) and \( F_7 \) (you will need to round, of course).
   (d) Show by induction that \( F_{3n} \) is even for any \( n \).
   (e) Show by induction that \( F_{5n} \) is divisible by 5 for any \( n \).

2. Arithmetic Sequences
   (a) Identify \( a_0 \) and \( d \) for each of the following sequences.
       i. 7, 12, 17, 22, ... 
       ii. 3.5, 5.25, 7, 8.75, 10.5, ... 
       iii. 8, 6.7, 5.4, 4.1, 2.8, ... 
   (b) Consider the arithmetic sequence with \( a_0 = 5 \) and \( d = 4 \).
       i. List the first 6 terms (\( a_0 \) to \( a_5 \)).
       ii. What is \( a_{30} \)?
       iii. What is the sum of the first 20 terms (\( a_0 + a_1 + \cdots + a_{19} \))?
   (c) Consider the arithmetic sequence with \( a_0 = 0 \) and \( d = 1 \).
       i. List the first 6 terms (\( a_0 \) to \( a_5 \)).
       ii. What is \( a_{30} \)?
       iii. What is the sum of the first 20 terms (\( a_0 + a_1 + \cdots + a_{19} \))?