Discrete Mathematics

Review for Test 2

NOTE: Write any probability as a decimal number rounded to four places.

1. Let \( A = \{1, 2, 3, 4\} \), \( B = \{4, 6\} \), \( C = \{3\} \) be three events for a uniform probability model with sample space \( S = \{1, 2, 3, 4, 5, 6\} \). Find each of the following.

(a) \( P(A) \)
(b) \( P(B) \)
(c) \( P(B \cap C) \)
(d) \( P(B \cup C) \)
(e) \( P(B^c) \)

2. In a certain probability experiment with events \( A \) and \( B \), we know \( P(A) = 0.6 \) and \( P(B) = 0.4 \). Find each of the following probabilities. If it is not possible in a particular case, say so.

(a) \( P(A \cup B) \) if you know \( P(A \cap B) = 0.2 \).
(b) \( P(A \cup B) \) if you know \( P(A \cap B) = 0.3 \).
(c) \( P(A \cup B) \) if you know \( P(A \cap B) = 0.4 \).
(d) \( P(A \cup B) \) if you know \( P(A \cap B) = 0.5 \).
(e) \( P(A \cup B) \) if you know \( A \) and \( B \) are mutually exclusive.
(f) \( P(A \cup B) \) if you know \( A \) and \( B \) are independent.

3. A five card poker hand is dealt from a regular deck.

(a) The total number of hands possible.
(b) The total number of hands consisting of all hearts.
(c) The probability of a flush.
(d) The probability of a full house.
(e) The probability of 1 pair.

4. A jar contains 5 fair dimes and 3 two-headed dimes. A dime is randomly removed and flipped.

(a) Build a tree to represent this experiment.
(b) What is the probability of flipping heads?
(c) What is the probability that you selected a fair coin, given that you flipped heads.
There are two identical jars on a table. Jar 1 contains 1 red marble and 1 blue marble, while jar 2 contains 4 red marbles and 1 blue marble. A jar is randomly selected and a marble randomly drawn from that jar.

(a) Build a tree to represent this experiment.
(b) What is the probability of drawing a red?
(c) What is the probability that you selected a jar 1, given that you drew a red.

5. A fair coin is flipped 3 times. The random variable $X$ counts the number of heads.

(a) What is the associated pmf?
(b) What is the associated cdf?
(c) What is $E[X]$?

6. A bad coin is a coin where the probability of heads is $1/3$. A bad coin is flipped 3 times. The random variable $X$ counts the number of heads.

(a) What is the associated pmf?
(b) What is the associated cdf?
(c) What is $E[X]$?

7. A committee consisting of 6 members is to be formed from a pool of 7 men and 9 women.

(a) What is the probability that such a committee is made up of only men?
(b) What is the probability that there are exactly 3 men on the committee?

8. A jar contains 5 red marbles and 1 blue marble. Marbles are successively drawn without replacement until a blue marble is obtained. What is the probability that the actual sequence is $RRRB$?

9. In a certain company, men with degrees are promoted with a probability of 0.8, while men without degrees are promoted with a probability of 0.25. There are 5 times as many men without degrees as there are men with degrees in the company. An HR rep randomly selects a person and notes that he has recently been promoted. What is the probability that he has a degree?

10. A fair coin is flipped 1000 times. Let $X$ count the number of heads. What is $P(X < 450 \text{ or } X > 550)$?

11. Approximate the number of primes in the set $\{1, 2, 3, 4, \ldots, 2000\}$

12. Prove that there are infinitely many prime numbers.

13. Prove that $\sqrt{2}$ is irrational.
14. Determine whether each of the following is prime, composite, or neither.

(a) 27  
(b) 29  
(c) 127  
(d) 129  
(e) 2377  

15. In testing the primality of 2377, what is the biggest prime you would need to consider?

16. Use Fermat’s little theorem with $a = 2$ to demonstrate that $p = 12$ is not prime.

17. Use Fermat’s little theorem with $a = 2$ to demonstrate that $p = 63$ is not prime (you’ll probably want to do some factoring first).

18. Use Euclid’s algorithm to find $(620, 725)$.

19. Use Euclid’s algorithm to find $(432, 1008)$.

20. Use Euclid’s algorithm to find $(141, 211)$.

21. Compute each of the following $(\mod 8)$

(a) 63  
(b) -42  
(c) 11  
(d) -5  
(e) 5  

22. Compute the first 20 terms the Fibonacci sequence $(\mod 3)$.

23. Compute the first 20 terms of the Fibonacci sequence $(\mod 4)$. 

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