1. Definitions: Be able to define each of the following.

(a) \((X, \tau)\) is second countable.
(b) \((X, \tau)\) is separable.
(c) \((X, \tau)\) is \(T_0, T_1, T_2, T_3, T_4\).
(d) \((X, \tau)\) is Regular.
(e) \((X, \tau)\) is Normal.
(f) \((X, \tau)\) is Connected (disconnected)
(g) The point \(p\) is a limit point for the set \(A\).
(h) \(A', \overline{A}, \text{Int}(A)\)
(i) \(D\) is dense in \(X\) (with topology \(\tau\)).
(j) The subspace topology.

2. Theorems: Be able to prove each of the following.

(a) 2nd countable \(\implies\) separable.
(b) Separable \(\nRightarrow\) 2nd countable.
(c) \(A\) is an open set if and only if \(\forall x \in A, \exists U \in \tau\; s.t. \; x \in U \subseteq A\).
(d) \(\text{Int}(A)\) is open.
(e) \(\overline{A}\) is closed.
(f) \(T_i \implies T_{i-1}\)
(g) If \(\tau\) is \(T_1\) and \(A\) is finite, then \(A' = \emptyset\).
(h) Prove that \(\mathbb{R}\) is 2nd countable.
(i) Prove that \(\mathbb{R}\) is separable.
(j) Prove that the line \(y = x + 3\) is a closed subset of \(\mathbb{R}^2\).
(k) Prove that \(T_i\) is hereditary.
(l) Prove that 2nd countable is hereditary.
(m) Prove that normal, regular, connected, separable are all non-hereditary (use counterexamples to do this).
(n) Prove that if \(S \subseteq T\) then \(S' \subseteq T'\).
(o) Prove \(\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}\)
(p) Prove $A \cup B = \overline{A \cup B}$

(q) Prove $Int(A \cap B) = Int(A) \cap Int(B)$

(r) Prove $(A \cup B)' = A' \cup B'$

3. Problems

(a) Describe three different bases for $\mathbb{R}^2$.

(b) Let $X = \{1, 2, 3, 4, 5\}$ and $\tau = \{\emptyset, X, \{1\}, \{1, 2\}, \{1, 2, 3\}\}$.
   - i. Find $A'$ for $A = \{2, 3\}$.
   - ii. Find $A'$ for $A = \{4, 5\}$.
   - iii. Find $A'$ for $A = \{3\}$.
   - iv. Find $Int(A)$ for $A = \{2, 3\}$.
   - v. Find $Int(A)$ for $A = \{1, 2, 3, 4\}$.

(c) Let $\tau$ be the Euclidean topology on $\mathbb{R}$.
   - i. Find $A'$ for $A = \{2, 3\}$.
   - ii. Find $A'$ for $A = (0, 2)$.
   - iii. Find $A'$ for $A = [0, 2]$.
   - iv. Find $A'$ for $A = \{1/2, 1/3, 1/4, \ldots\}$.
   - v. Find $A'$ for $A = \mathbb{Z}$.
   - vi. Find $A'$ for $A = \mathbb{Q}$.
   - vii. Find $Int(A)$ for $A = \{2, 3\}$.
   - viii. Find $Int(A)$ for $A = [0, 1)$.
   - ix. Find $Int(A)$ for $A = \mathbb{Q}$.

(d) Show $\overline{A \cap B} \neq A \cap B$

(e) Put a check in the box if the topology possesses the property.

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