Day 1.
Should know material

1. Sets of numbers – List each of the following sets
   (Do not forget the set symbols → {1, 3, 9} )
   a) natural numbers(N) or counting numbers: ____________________________

   Whole Numbers (W): ____________________________
   Integers (I, Z): ____________________________

   b) rational numbers:
   if we can write a decimal number as a fraction, then it must be a rational number. List some rational numbers that include almost all types.
   - 2, __________, __________, 3/11, __________, 0.13, __________

   c) irrational numbers:
   if we can not write a decimal number as a fraction, then it will be an irrational number. List some common examples that include almost all types

2. Other sets of numbers - List them.

   prime numbers: ____________________________________________________

   even, odd counting numbers: ____________________________________________

   Nonnegative integers: ________________________________________________

   Positive real numbers: ________________________________________________

3. True or False.
   If a line has a slope, then that slope will be positive when the line leans to the right. ________________

   If a line has a slope, then that slope will be negative when it leans to the left. ________________

   If a line is vertical, then it has a slope of zero. ________________

   All lines have some slope (a number). ________________

4. \( \sqrt{49} = \) __________  \( \sqrt{-25} = \) __________  \( \sqrt[3]{-64} = \) __________

5. Simplify by combining similar terms
   \( 2 + x + x + 5 = \) ________________
6. Factor
\[ 2x + 4y = \] 
\[ x^2 - 4x = \]

7. Find
\[ x + 2 \cdot 3 + x = \]

8. Closure of sets
We say set A is closed under addition if for any two numbers in the set (including the same number twice), the sum is also in the set.

A = \{ 0, 1 \} is closed under multiplication but is not under addition because even though 0 + 1 = 1 is in the set, 1 + 1 = 2 which not part of the set.

The set of natural numbers, integers, whole numbers are closed under addition. True or False.
The set of natural numbers, integers, whole numbers are closed under multiplication. True or False
\{ 0, -1, 1 \} is closed under addition. True or False
\{ 0, 1, -1 \} is closed under multiplication. True or False

What about under division and subtraction? Use the sets above – to determine which is commutative and/or associative

(subtraction) \( \) (division) \( \)


a) \( a + b = b + a \) is called the commutative law (property) of addition. \( \) (true or false)
a\( (bc) = (ab)c \) because of the associative law of multiplication. \( \) (true or false)

Assume that \( x \neq 0 \)
b) \[ x \div x = \] \[ x \cdot x = \]
c) Write down the distributive law: \[ \]

d) The additive identity: \( \) the multiplicative identity: \( \) the additive inverse of \( x \): \( \) (opposite)
the multiplicative inverse of \( x \): \( \) (reciprocal)
   a) \( x^4 \cdot x^5 = \) ___________     b) \( 2 + 3 \cdot 4 + 5 = \) ___________
   c) \( x^4 \div x^8 = \) ___________     d) \( x + 3 \cdot x + 5 = \) ___________
   e) \( (2x^3)^0 = \) ___________     f) \( (2x^3) \cdot (5x^4) = \) ___________

11. Solving Equations.
   a) \( 2 + 4x = -3 \rightarrow x = \) ________     b) \( \frac{x}{5} = \frac{1}{2} \rightarrow x = \) ________
   c) \( 2 + \frac{1}{2} + \frac{1}{4} = \) ___________     d) \( 2\frac{1}{3} \times 5\frac{1}{2} = \) ___________

   a) \( 2^4 = \) ________     b) \( 0^3 = \) ________     c) \( 8^0 = \) _______
   d) \( 4^{-1} = \) ________     e) \( -4^2 = \) ________

13. \( 2 + 3x + 8xy \) has three (factors or terms ?) ___________ (which one)

   \( 3xy \) has three (factors or terms ?) ___________ (which one)

14. Find the greatest common factor of 12 and 18. ___________

15. The least common multiple of 8 and 12 is ___________

16. What is the graph (shape) of the equation \( 2x - y = 4 \) ? _________________

18. If \( x = -2 \), then find \( y \) if \( y = x^2 - 2x - 2 \). _______________
19. Construct a rectangular coordinate system and label each of the four quadrants as well as the axes.

20. Sketch the graph of $2x + 3y = 12$.

21. Find

\[
\frac{1}{2} + \frac{1}{4} = \underline{\quad} \\
\frac{1}{2} \times \frac{1}{4} = \underline{\quad} \\
\frac{1}{2} \div \frac{1}{4} = \underline{\quad} \\
\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \underline{\quad}
\]

22. Which is larger

the smallest whole number   the smallest counting number   the first prime number

None, they are all equal   not enough information

23. Which is even

the product of the $1^{st}$ 2 prime numbers   the sum of the $1^{st}$ 2 odd numbers

Not enough information   they both are even

24. As $n$ gets larger and larger (as $n \to \infty$)

what happens to the values of $(1 + 1/n)^n$?
Math 1302  College Algebra

Sets Of Numbers

\[ \mathbb{N} = \{1, 2, 3, 4, \ldots\} \] is called the set of \[ \ldots \] Let \( x \) be one of these numbers. Find all natural number solutions of the following equations.

\[ 2x + 4 = 6 \Rightarrow x = \underline{\quad} \quad \quad \quad x - 2 = 2 \Rightarrow x = \underline{\quad} \]

\[ 2x + 4 = 4 \Rightarrow \underline{\quad} \]

We can use the set of natural numbers to describe the

1) The number of times a student will have to enroll in a required class before the class is passed with a grade of C or above.
2) The number of jobs a currently employed individual has held during his/her lifetime.

NOTE:

a) The set of prime numbers: a natural number that is greater than one and is divisible (evenly) only by one and itself
   \[ \{2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots\} \]

b) The smallest natural number is one, there is no largest

\[ \mathbb{W} = \{0, 1, 2, 3, \ldots\} \] is called the set of \[ \ldots \] Let \( x \) be one of these numbers. Find all whole number solutions of the following equations.

\[ x - 4 = 6 \Rightarrow x = \underline{\quad} \quad \quad \quad x + 5 = 5 \Rightarrow x = \underline{\quad} \]

\[ x + 6 = 2, \Rightarrow x = \underline{\quad} \]

We can use whole numbers to describe the

1) number of days without an accident
2) the number of friends that you can wake up at 2:00 AM just to say hello
3) The number of problems you will answer correctly on the first exam.

Note:

1) Zero (0) is the smallest whole number and is the only whole number that is not a natural number.
\( \mathbb{Z} = \{ ..., -3, -2, -1, 0, 1, 2, 3, ... \} \) is called the set of __________

Let \( x \) be one of these numbers. Find all integer solutions of the following equations.

\[
\begin{align*}
x - 4 &= -6 \Rightarrow x = \_\_\_\_ \\
2x + 1 &= 1 \Rightarrow x = \_\_\_\_ \\
3x &= 1 \Rightarrow x = \_\_\_\_
\end{align*}
\]

1) A game costs $1 to play. If you lose, you lose your dollar and if you win, you win $5. Record the amount won.
2) The number of yards gained by a football player

Note: Now we can introduce the idea of positive and negative numbers

1) the set of positive integers \( \{ 1, 2, 3, ... \} \), 2) the set of negative integers: \( \{ -1, -2, -3, ... \} \)
3) zero is neither positive nor negative
4) the set of nonnegative integers: \( \{ 0, 1, 2, 3, 4, ... \} \)

Other Sets

The set that contains all real numbers that can be written as fractions (ratio of integers) is called the set of ________________

We describe this set in what is called set-builder notation: \( \mathbb{Q} = \{ \frac{a}{b} \mid a \text{ is an integer and } b \text{ is a nonzero integer} \} \)

For example: \( -2/3, \ 7/11, \ 2/9, \ -6/2=3, \ 4/1=4, ... \)

Note:

Rational numbers include simple fractions like \( 2/3, \ -7/11, \ 1/4, \ 11/5, ... \) that can not be reduced
but also numbers that can be labeled as integers \( \rightarrow 3, -2, -7, 0, 2, ... \) they can all be written as fraction by writing them with a denominator of one \((1)\).

Note:

Rational numbers consists of simple fractions and integers
integers consist of negative integers and whole numbers
whole numbers consist of natural numbers and zero

All rational numbers can be written as fractions and also as decimals that are either
terminating: \( 0.12, \ 3.1114, \ 5.000000007, \)
or repeating block: \( 3.12121212... , \ 4.111111..., \ 6.123412341234... \)

6
Real numbers that are not rational numbers
( decimal numbers that can not be written as fractions)

are called ________________________________

( this means the real numbers are made up entirely of rational and irrational numbers)

This implies that irrational numbers can not be written in a “nice” decimal pattern (not terminating) They do include the following numbers →

2.1010010001000100000...
423.1223331122233331122223333...

More commonly we also include the following as irrational numbers

\[ \pi = 3.14... \]
\[ \sqrt{2} = \ldots \]
\[ \sqrt{5} = \ldots \]
\[ e = \ldots \]

as n gets larger and larger what happens to \( \left(1 + \frac{1}{n}\right)^n \)?

<p>| | | | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>1000</td>
<td>2000</td>
<td>5000</td>
<td>10000</td>
<td>20000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.25</td>
<td>2.48832</td>
<td>2.593742</td>
<td>2.653298</td>
<td>2.691588</td>
<td>2.704814</td>
<td>2.711517</td>
<td>2.715569</td>
<td>2.716924</td>
<td>2.717603</td>
<td>2.718214</td>
<td>2.718255</td>
<td>2.718268</td>
<td>2.718275</td>
</tr>
<tr>
<td>500</td>
<td>1000</td>
<td>2000</td>
<td>5000</td>
<td>10000</td>
<td>20000</td>
<td>50000</td>
<td>100000</td>
<td>200000</td>
<td>500000</td>
<td>1000000</td>
<td>2000000</td>
<td>2.715569</td>
<td>2.71692393</td>
<td>2.717603</td>
</tr>
</tbody>
</table>

Closure:

We say a set is closed under a given operation if when you perform the operation on any two numbers of that set, the result is a third number that is also a member of that set.

Example:

Time on a clock: Add any amount of time to a clock – do you still get a time on the clock.

1) natural numbers are closed under addition and multiplication: 3 + 7 = 10 all natural numbers, 4 * 5 = 20 all natural numbers

2) What other sets are closed under addition and multiplication:

3) What sets are closed under subtraction? Under division?

This concludes a brief review of the sets that make up the set of real numbers. If we wish to solve equations of the form \( x^2 + 4 = 0 \), we have to extend the idea of numbers to a larger set – The Set of Complex Numbers.

For now we will only look at the set of real numbers. Later on, we will look at the set of complex numbers.
Practice Problems #1

1. Identify each set that each of the following numbers can be classified as being a part

<table>
<thead>
<tr>
<th>Number</th>
<th>natural #</th>
<th>whole #</th>
<th>Integer</th>
<th>Nonnegative</th>
<th>Rational</th>
<th>Irrational</th>
<th>Real #</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.232323...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sqrt{3})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.121122111222...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sqrt{-4})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Find the sum of the first four prime numbers. _______________

3. Find the product of the first three prime numbers. ____________

4. How many integer solutions does the equation \(x^2 - 9 = 0\) have? ________

5. How many real number solutions does the equation \(2x - 4 = 0\) have? __________

6. How many natural number solutions does the equation \(3x + 9 = 0\) have? ________

7. True or False.
   a) the set of integers are closed under division. __________
   b) the set of irrational numbers are closed under multiplication. __________
   c) the set of rational numbers are closed under division. __________

8. We can write a natural number that is not prime and greater than one (composite #) as a product of prime numbers.
   For example: \(6 = 2 * 3\). Write 42 as a product of prime numbers. This is called the prime factorization of 42.
   \[42 = \] ________________

9. What is the smallest positive real number? ________________
   What is the smallest nonnegative integer? ________________

10. All rational numbers can be written as a fraction. Write each of these rational numbers as fractions.
    a) 0.66666...  
    b) 2.1111...  
    c) 0.232323...
Properties of Real Numbers

commutative law of addition

\[ a + b = \text{________} \]
\[ 3 + 2 = 2 + 3 \]
\[ 5 + 2 = \text{________} \]
\[ (91 + 17) + 9 = \text{________} \]

commutative law of multiplication

\[ ab = \text{________} \]
\[ 12 \cdot 7 = 7 \cdot 12 \]
\[ 6 \cdot 3 = \text{________} \]
\[ (8 \cdot 7) \cdot 5 = \text{________} \]

associative law of addition

\[ a + (b + c) = \text{________} \]
\[ (5 + 15) + 17 = \text{________} \]
\[ (93 + 16) + 4 = \text{________} \]

associative law of multiplication

\[ (a \cdot b) \cdot c = \text{________} \]
\[ (8 \cdot 5) \cdot 13 = \text{________} \]
\[ (17 \cdot 4) \cdot 5 = \text{________} \]

Distributive law of multiplication over addition: this property combines both the addition and the multiplication properties

\[ a(b + c) = \text{________} \]
\[ -2(x - 3) = \text{________} \]
\[ -(a - b) = \text{________} \]
\[ (a + b)(x + y) = \text{________} \]

Commutative and the associative laws do not hold true with subtraction and division – do they?

ex. \[ 2 - 4 = 4 - 2 \? \text{________} \]
\[ 12 \div 6 = 6 \div 12 \? \text{________} \]

ex. \[ (12 - 4) - 2 = 12 - (4 - 2) \? \text{________} \]
\[ (16 \div 8) \div 2 = 16 \div (8 \div 2) \? \text{________} \]
Just in case you think multiplication is always commutative – think of multiplication of matrices.

\[
\begin{bmatrix}
1 \\ 2 \\ 3
\end{bmatrix} \cdot [4 \quad 5 \quad 6] = [4 \quad 5 \quad 6] \cdot \begin{bmatrix}
1 \\ 2 \\ 3
\end{bmatrix}
\]

Is the right side equal to the left side?

Other Properties of Real Numbers:

Identity Property of Addition:
if x is any real number, then \( x + 0 = x \), 0 is called the additive identity → \((-3/5) + 0 = \)_______

Identity Property of Multiplication:
if x is any real number, then \( x \cdot 1 = x \), 1 is called the______________________ → \((1.7) \cdot 1 = \)_______

Inverse Property of Addition:
if x is any real number, then \( x + ( - x ) = 0 \),
x and -x are called opposites or additive inverses → \((-3) + ____ = 0\)

Inverse Property of Multiplication:
if x and y are any real numbers with \( x \neq 0 \) and \( y \neq 0 \)
and \( x \cdot y = 1 \), then x and y are called__________________________. → \((-4) \cdot ____ = 1\)
Practice Problems#2:

1. Use the commutative property of addition to complete: \( 3 + 4 = \) 

2. Give me an example of the associative law of multiplication. 

3. What rule of real numbers is being used here \( 3(4 + x) = 3(4) + 3(x) \)? 

4. True or False. 
   a) \((3 + 4) + 5 = (4 + 3) + 5\) is an example of the associative law of addition. 
   b) zero has no multiplicative inverse. 
   c) \(x = 3\), then the additive inverse of \(x\) is \(-3\). 
   d) all whole numbers have an additive inverse. 
   e) 1 is called the multiplicative inverse. 
   f) Division and subtraction are not commutative in the set of real numbers. 

5. Use parenthesis to complete each statement so that the right and left side equal each other 
   a) \(13 + 8 - 5 = 16 \) → 
   b) \(4 \cdot 3 + 2 - 10 = 14 \) → 
   c) \(12 \div 6 - 2 = 3 \) → 

6. Find the missing value so that the right side of the equation is equal to the left side. 
   a) \(8(3 + 4) = 8(\quad) + 8(\quad) \) → 
   b) \((\quad)(1 + 8) = 7(1) + 7(9) \) → 
   c) \(3 + 5 = 5 + (\quad) \) → 
   d) \(12(8) = 12(5) + 12(\quad) \) → 

Greatest Common Factor

Factors – Divisors

Find the factors of 48: Find the factors of 124:

Find the factors of 68:

GCF: greatest common factor of a set of numbers
the largest number that will divide into every member of the given set

ex. Find the greatest common factor of 8 and 12

1) method one: the factors(divisors) of 8 are 1, 2, 4, 8 the factors of 12 are 1, 2, 3, 4, 6, 12
the largest factor they have in common is 4 → GCF(8, 12) = 4

Find the GCF of 20 and 16

Factors of 20: ________________ Factors of 16: ________________

2) method two: prime factorization (the process of writing a number as a product of its prime factors)

8 = 4 • 2 = 2 • 2 • 2 = 2^3
12 = 6 • 2 = 2 • 3 • 2 = 2^1 3^1

Select the common base(s) with the smallest corresponding exponent → 2^2 = 2 = GCF
if more than one base, the product

Find the GCF of 24 and 20

Prime Factorization of 24: __________ Prime Factorization of 20: ____________
ex. Find the GCF of 12 and 20 by the first method:

ex. Find the GCF of 8, 20, and 30 by using the second method:

Third Method (Simple)

Guess and then decide if correct – if not correct, correct. Continue until correct.

Find the GCF of 24 and 64.

Find the GCF of 120 and 200.

There is a 4th method:

GCF (12, 8) → Divide 8 into 12 and use the remainder 4 →
    divide 4 into 8 → the remainder is 0
    so 4 is the GCF of 8 and 12

WE find the GCF when the remainder is zero.

GCF (36, 24) → divide 24 into 36 and use the remainder 12 →
    divide 12 into 24 → the remainder is zero →
    so 12 is the GCF

Find GCF (40, 24). ___________ Find the GCF of 24 and 30. ___________

Find the GCF of 248 and 128.
Word Problem examples

a) Pencils are sold in bundles of 20, 50, and 100. How should pencils be packaged so that when an order is received, the order will be filled without breaking up the package? (Select as large a package as possible to be able to fill an order—a bundle)

   If the packages were in groups of 5: a bundle of 20 requires 4 packages of 5, 50 requires 10 packages of 5,...

b) Students are assigned to small groups which will be combined to form larger groups to tour the campus. A tour guide will then take groups of 24, 36, or 60. The destination will be used to determine which size group will be taken. How should the students be grouped so that when the actual size of the tour is determined the smaller subgroups can be grouped together to add up to 24, 36, or 60 without having to break-up the subgroups? (Make your groups as large as possible)
Least Common Multiple

Multiples:

Find the multiples of
7: _______________________  12: _______________

20: _______________________

LCM: least common multiple
the smallest number that is divisible by every member of a given set

ex. Find the LCM of 20 and 15

1) method 1
multiples of 20: 20, 40, 60, 80, 100,.... multiples of 15: 15, 30, 45, 60, 75, 90, ...
the least (smallest) multiple they have in common is 60. So, LCM (15, 20) = 60

Find LCM (15, 25). _________

2) method 2 (prime factorization)

20 = 4 \cdot 5 = 2 \cdot 2 \cdot 5 = 2^1 \cdot 5^1
and
15 = 5 \cdot 3 = 3^1 \cdot 5^1

Select all distinct bases (list them once each) with the largest corresponding exponent
The product of these numbers will give you the LCM.

\[ 2^1 \cdot 3^1 \cdot 5^1 = 4 \cdot 3 \cdot 5 = 60 = \text{LCM} (20, 15) \]

Find LCM (70, 28). _________
3) **Alternate Method**: when the GCF is known.

GCF (15, 20) = 5  
multiply original numbers → 15 \cdot 20 = 300  
divided the result by the GCF → 300 ÷ 5 = 60 = your answer.

Find LCM (20, 32). ____________

**Other examples:**

Find LCM (20, 15) = ________

Find LCM (2, 3, 5) = ____________

Find GCF and LCM of 150 and 240. ________________  ________________

**Examples using word problems**

a) A walker takes five minute to walk around a track while a runner takes two minutes to travel around the same track. They start at the same time and walk in the same direction. How often will they meet at the starting point?

b) Two Clocks ring at different times

First one beeps every 20 seconds, the second one will beep every 35 seconds. How often will they beep at the same time? Assume that they both were started at the same time.
Absolute Values

Absolute Value of a real number:
Let $x$ be a real number. The absolute value of $x$, written $|x|$, is defined by

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Notes:
You can think of the absolute value as an undirected distance (positive or zero) from the origin to the number (the point representing the number).

An absolute value makes the number (or the result) nonnegative. You can have zero or positive but no negative results.

ex. $|6| = ________$, $|-2| = ________$

$-|-4| = ________$, $|x| = ________$ if $x$ is a natural number

(say it in words)

ex. If $|c| = 5$, then where is $c$ on the number line?

Other examples:

1) $|3 - 5| = \underline{\text{__________}}$ (without combining the numbers)  
2) $|1 + \sqrt{3}| = \underline{______}$

3) $|3 - \pi| = \underline{\text{__________}}$  
4) $|\sqrt{10} - 3| = \underline{______}$

5) $|-2| - |-3| = \underline{______}$  
6) $|4| - |-6| = \underline{______}$

7) $x / |x| = \underline{______}$ if $x \neq 0$  
8) $|4| / |-8| = \underline{______}$

9) $|x^2| = \underline{\text{__________}}$  
10) $|-x| = \underline{______}$
Properties of Absolute Values:

1) $|c| \geq 0$
   
   ex. $|12| = 12 > 0$  
   ex. $|-2/3| = 2/3 > 0$  
   ex. $|0| = 0 \geq 0$

2) $|-c| = |c|$
   
   ex. $|-4| = |4|$

3) $|a \cdot b| = |a| \cdot |b|$
   
   $|4 \cdot 5| = |4| \cdot |5|$

   3.2) $|x^2| = (x)^2$
   
   This will be discussed later if at all

3.3) $|x^2| = x \cdot x$

4) $|a/b| = |a| / |b|$
   
   $|-12/4| = |-12| / |4|$

5) $|a + b| \leq |a| + |b|$
   
   $|-2 + 5| \leq |-2| + |5|$

   $|-4 - 3| \leq$
Problem set #3

1. Find the GCF of 12 and 2
Inequalities

< : means less than
4 < 6 → we say 4 is less than 6

≤ : means less than or equal to
5 ≤ 7 → we say 5 is less than or equal to 7,
6 ≤ 6 We say 6 is less than or equal to 6

> : means greater than
-2 > -4 → We say that -2 is greater than – 4

≥ : means greater than or equal to
12 ≥ 10 → we say 12 is greater than or equal to 10
8 ≥ 8 → we say that 8 is greater than or equal to 8

True or False

4 < -4 ________ -2 ≤ -2 _____ 3/11 > 2/9 ________

0.01 < 0.0091 ________ | -3 | < | -8 | ________

x² ≤ x , if |x| < 1

We can use inequalities to represent a set of numbers on a number line – (also, interval notation)

x < 4

x ≥ 2

-4 < x ≤ 2

0 ≥ x ≥ 4
**Algebra: Basic Terminology**

**literals, variables**: we use letters to represent quantities (real numbers) – we normally use lower case letters

\[ x, y, z, \text{ most common} \]

**Algebraic Expressions**: use arithmetic operations \((+, -, \cdot, \div)\) with variables

\[ x + 2y \quad 2a - xy \quad \frac{1}{3} + 0.5c + a \]

**Evaluate Algebraic expressions**: replace variable with given values and simplify

if \( x = -2, y = 3, \text{ and } z = 0 \), then find

\[ x^2 + y^0 = \quad \quad xz^2 = \quad \quad \frac{x - y}{xy} = \]

**Terms and Factors**:

**terms** – literal expressions being added or subtracted \(\rightarrow\)

\[ x + 2xy + 2 \rightarrow \text{has three terms} \quad 2x - \frac{1}{2} \rightarrow \text{has two terms} \quad 2xy \rightarrow \text{has 1 term} \]

**factors** – literal expressions being multiplied

\[ 2xy \rightarrow \text{has 1 term but three factors} \quad \frac{1}{4}x^2yz \rightarrow \text{has 1 term with four factors} \]

**Coefficients and Constants**: 

an algebraic expression with no variable is called a constant term. \(x + 2y + 3\)

\( + 3 \) is a constant (constant term), \( x + 2y + c \rightarrow c \) is the constant term

The constant in front of a product of variables is called the coefficient. \(\rightarrow\)

\( 2x - y \) has 2 and -1 as coefficients

\( bx + cy \) has coefficients \( b \) and \( c \)
Like terms, similar terms:
terms that have the same variables and corresponding exponents, coefficients may be different

\[ 2xy + 3xy \text{ are similar terms; } 2x^3y - 5xy^3 \text{ are not similar} \]

\[ \sqrt{2x} - 5 \sqrt{2x} \text{ are also similar} \]

demo

Find the sum of \( 2x + (3x - 2y) \).

Find the difference of \( 3x - (2 - 2x) \).

Use the distributive property to simplify. \( 2(x - 2) - 3(x + 2) = \) 

Find

\[ 2 + 3x = \] \[ 5x - 2 = \]

\[ x + 2 \cdot x + 3 = \]
Exponents

Integer Exponents

$x^n$: exponential notation, 

$x$ is called the _________ and $n$ is called the _________ or the _________

and $x^n = x \cdot x \cdot x \cdot \ldots \cdot x$ (a total of $n$ x’s) – this is the expanded notation

Examples:

$2^5 = \ldots$ \hspace{1cm} $( -4)^3 = \ldots$

$- (4)^3 = \ldots$ \hspace{1cm} $-3 \cdot 3 \cdot 3 \cdot 3 = \ldots$

$-4^2 = \ldots$ \hspace{1cm} $( -2/3)^4 = \ldots$

ex. $0.01^4 = \ldots$

Note: $( -x)^n = -x^n$ only if $n$ is odd \hspace{1cm} $\rightarrow$ \hspace{1cm} $( -2)^3 = ( -2)^3$ but $( -2)^4 \neq -2^4$

Def. If $x$ is a real number not equal to zero, then $x^0 =$

$(4)^0 = \ldots$ \hspace{1cm} $( -2)^0 = \ldots$ \hspace{1cm} $( -c)^0 = \ldots$ for $c \neq 0$

$(0)^4 = \ldots$

What about $- (4)^0 = -4^0 = \ldots$

So, $-3^0 = \ldots$ and $-(-2)^0 = \ldots$. What is $0^0 = \ldots$
Def. If \( n \) is a natural number, then \( x^{-n} = \) \phantom{} \phantom{} \phantom{.}.

This gives us a way to work with negative exponents in terms of natural numbers.

ex. Find

\[
2^{-2} = \phantom{} \phantom{} \phantom{} \phantom{} \phantom{}
\]

\[
4^{-3} = \phantom{} \phantom{} \phantom{} \phantom{} \phantom{}
\]

\[
8^{-1} = \phantom{} \phantom{} \phantom{} \phantom{} \phantom{}
\]

What about

\[
\left(\frac{1}{4}\right)^{-2} = \phantom{} \phantom{} \phantom{} \phantom{} \phantom{}
\]

\[
\left(\frac{1}{2}\right)^{-3} = \phantom{} \phantom{} \phantom{} \phantom{} \phantom{}
\]

\[
\left(\frac{3}{4}\right)^{2} = \phantom{} \phantom{} \phantom{} \phantom{} \phantom{}
\]

Also,

\[
\left(-2\right)^{-2} = \phantom{} \phantom{} \phantom{} \phantom{} \phantom{}
\]

\[
\left(-\frac{2}{3}\right)^{-2} = \phantom{} \phantom{} \phantom{} \phantom{} \phantom{}
\]

\[
\left(-\frac{4}{5}\right)^{-3} = \phantom{} \phantom{} \phantom{} \phantom{} \phantom{}
\]

\[
24
\]
Properties of Exponents.
Let \( x \) and \( y \) represent real numbers and let \( m \) and \( n \) be natural (counting) numbers.

1. \( x^n \cdot x^m = \) __________
   
   \( x^4 \cdot x^5 = \) __________
   \( x^5 \cdot x = \) __________

2. \( x^n \div x^m = \) __________
   
   \( x^8 \div x^4 = \) __________
   \( x^{12} / x^{20} = \) __________

3. \( (x^n)^m = \) __________
   
   \( (x^3)^2 = \) __________
   \( (x^6)^3 = \) __________

4. \( (x \cdot y)^n = \) __________
   
   \( (2x)^3 = \) __________
   \( (4xy)^3 = \) __________
   
   \( (x^2y^3)^2 = \) __________
   \( (2xy^3)^3 = \) __________

5. \( (x \div y)^n = \) __________
   
   \( (x/y)^4 = \) __________
   \( (x^2 / y^3)^3 = \) __________
Problem Set # 4
Other examples – on exponents

1)  \((2x^3y) \cdot (-4x^2y^4) = \) \[\text{_______________}\]

2)  \((2xy^2)^2 \cdot (3x^2y)^3 = \) \[\text{_______________}\]

3)  \(\frac{4xy}{12xy^6} \div \frac{-2y^3}{x^2y} = \) \[\text{_______________}\]

4)  \(\left(\frac{-2x}{y^2}\right)^2 \div \left(\frac{3xy^3}{-2y^3}\right)^2 = \) \[\text{_______________}\]

Since we have a definition for negative exponents – how would we work problems with negative exponents.

1.  \((x^{-2})^{-4} = \) \[\text{_______________}\]

2.  \((x^{-3}y^4)^2 = \) \[\text{_______________}\]

3.  \((x^3)^2 = \) \[\text{_______________}\]

4.  \((2x^{-2})^2 = \) \[\text{_______________}\]

5.  \((2x^3y^0) \cdot (3x^{-1}y) = \) \[\text{_______________}\]

6.  \(\frac{2x^{-1}y}{3x^{-4}y^2} = \) \[\text{_______________}\]
3. \[ \left( \frac{-4x^{-2}y^3}{12x^{-2}y^{-2}} \right)^2 = \]

4) \( ( -4/3 )^{-3} = \) \( ( \frac{-4x^{-1}}{y^3z} )^{-2} = \)

Since the rules work with integral exponents – do they work with rational exponents? Yes

1) \( x^{2/3} \cdot x^{7/3} = \) \( x^{1/3} \cdot y \cdot x y^{2/3} = \) \( x^{-2/3} )^{-6} = \)

4) \( ( -5x^{1/2}y)^{1/2} = \)

5. \( \left( \frac{3x^{1/2}y^{-1/4}}{1} \right)^{-2} = \)

6. \( \frac{4x^{-1/2}y^{-1/3}}{8xy^{-2/3}} = \)
More Examples of exponent problems –

Find

\((-2xy^3)^0 = \underline{\text{__________}}\) if neither x nor y = 0 \hspace{1cm} 18^0 = \underline{\text{__________}} \hspace{1cm} -4^0 = \underline{\text{__________}}

\[0^4 = \underline{\text{__________}} \hspace{0.5cm} 0^2 = \underline{\text{__________}} \hspace{0.5cm} 0^0 = \underline{\text{__________}} \hspace{0.5cm} -4^2 = \underline{\text{__________}} \hspace{0.5cm} -4^{-2} = \underline{\text{__________}}\]

Also, if x = -1 and y = -2 and z = 0 find

1) \(x^y = \underline{\text{__________}}\) \hspace{1cm} 2) \(y^0 = \underline{\text{__________}}\) \hspace{1cm} 3) \(-x^2 = \underline{\text{__________}}\) \hspace{1cm} 4) \((x - y) ÷ x + y = \underline{\text{__________}}\)
**Scientific Notation:**

A real number can always be written in decimal notation. If the number is written in the form 

\[ r.pppp \ldots \times 10^n \], where \( 1 < r \leq 9 \) and \( n \) is an integer, then we say the number is in scientific form (notation).

Write in scientific notation:

a) \( 235.2 \rightarrow \) __________________

b) \( 0.0034 \rightarrow \) __________________

c) \( 4000 \rightarrow \) __________________

Change from scientific notation to standard form (\( 2.3 \times 10^{-3} = 0.0023 \))

a) \( 2.009 \times 10^2 = \) ______________

b) \( 1.23 \times 10^{-4} = \) ______________

Perform the operation and leave result in scientific notation.

a) \( (6 \times 10^3) \times (8 \times 10^7) \div (1.2 \times 10^3) = \) ______________

b) \( (4 \times 10^{-2}) \times (8 \times 10^{-4}) \times (6 \times 10^{-6}) = \) ______________
Radicals

Roots: square roots, cube roots, 4th roots, ...

We say $c$ is a square root of $b$ if $c^2 = b$.

Square roots: of $49 \implies \_\_\_\_\_\_\_\_$ square roots of $81 \implies \_\_\_\_\_\_\_\_$

We say $c$ is a cube root of $b$ if $c^3 = b$

Cube roots of $64 \implies \_\_\_\_\_\_\_$ cube roots of $-27 \implies \_\_\_\_\_\_\_$

We say that $c$ is an $n$th root of $b$ if $c^n = b$

Find the square roots of $25 \implies \_\_\_\_\_\_\_$ Find the fifth roots of $-32 \implies \_\_\_\_\_\_\_$

Find the 4th roots of $(-16) \implies \_\_\_\_\_\_\_$ Find the 4th roots of $81 \implies \_\_\_\_\_\_\_$

Because some values have more than one $n$th root, we choose to bring up the idea of principal $n$th roots and avoid some of the confusion.

Note:

If a number has an $n$th root, then the following statements are true

If $n$ is even (square roots, fourth roots, sixth roots, ...), then we have two $n$th roots

If $n$ is odd (cube roots, fifth roots, ..., ) , then we have only one $n$th root.

To avoid the confusion of having two roots let’s define the roots by the use of “radicals” and the phrase “principal (nth)root”
Define Principal nth roots of $x$: $\sqrt[n]{x}$; 
n is called the ___________________, $x$ is the _____________

if $x > 0$, then $\sqrt[n]{x} > 0$ \Rightarrow

\[ \sqrt[4]{625} = ______ , \quad \sqrt[3]{64} = ______ \]

if $x < 0$ and $n$ is odd, then $\sqrt[n]{x} < 0$ \Rightarrow

\[ \sqrt[3]{-27} = ______ , \quad \sqrt[3]{128} = ______ \]

if $x < 0$ and $n$ is even, then $\sqrt[n]{x}$ has no real value \Rightarrow

\[ \sqrt[2]{-4} = ______ , \quad \sqrt[4]{-1} = ______ \]

Note: Because square roots are so common (most common to us) –

we usually leave off the index \rightarrow $\sqrt[2]{x} = \sqrt{x}$

ex. Find each of the following nth roots

1) $\sqrt[2]{64} = ____$  
2) $\sqrt[3]{-8} = ______$  
3) $\sqrt[2]{-25} = ______$

4) $\sqrt[4]{x^8} = ____$  
5) $\sqrt[3]{x^6y^4} = ______$  
6) $\sqrt[3]{8x^3y^{12}} = ______$

True or False:

$\sqrt{x^2} = x$ ___________________ So, $\sqrt{(-2)^2} = ______$

However, if we assume that $x$ is a positive real number, then $\sqrt{x^2} = x$

but in general $\sqrt{x^2} = |x|$.
Define 
\[ x^{1/n} = \sqrt[n]{x} \]

Examples of \( x^{1/n} \)

1) \( 8^{\frac{1}{3}} = \) ________________

2) \( 16^{\frac{1}{4}} = \) ________________

3) \( -25^{\frac{1}{5}} = \) ________________

4) \( (\cdot 27)^{\frac{1}{3}} = \) ________________

5) \( (-9)^{\frac{1}{3}} = \) ________________

6) \( (16)^{-\frac{1}{4}} = \) ________________

Define 
\[ x^{m/n} = (\sqrt[n]{x})^m \quad \text{or} \quad x^{m/n} = \sqrt[n]{x^m} \]

Now we can use fractional exponents:

\[ 16^{\frac{1}{5}} = \), -9^{\frac{3}{2}} = \), -16^{\frac{1}{2}} = \]

ex. \( (16x^{2/3}y^{9})^{1/4} = \) ________________

ex. \( (-8x^{6/9})^{2/3} = \) ________________

ex. \( (4x^{-4}y^{8})^{-\frac{3}{5}} = \) ________________
More on radicals:

While we can simplify quantities that are perfect squares, perfect cubes,…

such as \( \sqrt[3]{64} = \) _____ or \( \sqrt[4]{x^8y^{12}} = \) _____

What about quantities that are not “perfect”

\( \sqrt{20} = \) _____ \( \sqrt[3]{16} = \) _____

\( \sqrt[3]{x^3} = \) _____ \( \sqrt[3]{16x^5} = \) _____

We can approximate their values by using a calculator

Properties:

1) \( \sqrt{xy} = \sqrt{x} \cdot \sqrt{y} \)

ex. \( \sqrt[2]{4x^2} = \) ______________

2) \( \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}} \)

ex. \( \sqrt[4]{\frac{9x^4}{4y^2}} = \) ______________
We also need to be able to factor the coefficients in terms of their prime factors to simplify radicals.

ex. Write 42 in factored form (as a product of prime numbers) = ______________

ex. What is the prime factorization of 24? 24 = ______________
We say a radical is in simplest form if (radical coefficients must be in prime factored form) if the following three rules are satisfied:

1) The index is smaller than all of the exponents inside the radical

\[ \sqrt[3]{x^6} = \ldots \quad \sqrt[3]{x^2} = \ldots \quad \sqrt{25} = \ldots \]

\[ \sqrt[5]{x^8} = \ldots \quad \sqrt[4]{16x^5y^2} = \ldots \]

2) There is no common factor between the index and all of the exponents inside the radical

\[ \sqrt[4]{x^3} = \ldots \quad \sqrt[6]{8x^3y^3} = \ldots \quad \sqrt[4]{2x^2} = \ldots \]

3) There can be no radical in the denominator or no denominator inside the radical.

ex. \[ \frac{2}{\sqrt{2}} = \ldots \]

ex. \[ \frac{4x}{\sqrt{4x}} = \ldots \]

ex. \[ \ldots \]

ex. \[ \ldots \]
More examples.

a) \( \sqrt[4]{\frac{9x}{4y}} = \)

b) \( \sqrt[3]{\frac{9x}{4y^2}} = \)

Other radical problems:

\( 4\sqrt{9} = \) ________  \( 9 + \sqrt{8} = \) ________  \( 4 + 2\sqrt{9} = \) ________

\( 2\sqrt{3} - 3\sqrt{12} = \) ________  \( \sqrt{5} \cdot \sqrt{20} = \) ________

\( \sqrt[3]{4x} \cdot \sqrt[3]{2x^2} = \) ________  \( \sqrt{2} \cdot \sqrt{2} = \) ________

\( 3\sqrt[3]{8} = \) ________
Still more on Radicals:

a)

b)

c)      d)
Problem set #6
Polynomials

Sum of literal expressions in which each term consists of a product of constants and variables with the restriction that each variable must have a nonnegative integer exponent.

2x, 3x²y - 4, 1 + x + 3x², 5 - 3xy + y⁹, .... ➔ What about ______________ or ______________?

Special types of polynomials

if one term: ___________  if two terms: _______________  if three terms: ___________

How many terms does each of the polynomials have?

2xy 3 + 2xy + x⁹ + y² 1 + x + x²

_____________ __________________________ __________________

Def. (degree)

monomials: 3x ==> _________ 2x⁵ ==> _________ ½ x⁵ y³ ==> _________

binomials: 2x - 1 ==> _________ x + y ==> _________ 3x²y - 4x³y² ==> _________

other polynomials: x - 2xy + y³ ==> _________ x⁹ - 2xy + x⁶y⁵ ==> _________
Basic operations of polynomials: sum – difference, products - quotients

Find (sum and differences)

a) \((2xy - 4x) + (3xy - 2x)\)  ==> _____________

b) \((3x^2 - 2x + 3) - (2x^2 - 4x - 5)\)  ==> __________

Products

a) \(2(x - 3y)\) = _______________  b) \(x(x + 2y)\) = _______________

c) \(3x^3(3xy - 2x^4y)\) = _______________

d) \((2x - 3y)(4x + 2y)\) = _______________

Def. The process of writing a polynomial as a product of other polynomials of equal or lesser degree is called ___________________

Recall: GCF

Find \(\text{GCF} (20, 36)\) = ___________ \(\text{GCF} (xy, x)\) = ___________ \(\text{GCF}(2xy, 6y^2)\) = ___________

\(\text{GCF}(12x^3y^6, 8x^4y^2)\) = ___________
Special Products and Factoring

1) greatest common factor:
   \( x(y + x) = \) 

\[ 2(3x - y) = \]  
\[ 4x(5 - 3x) = \] 

Factor GCF:
\[ 2 - 12x = \]  
\[ x(y - 1) - y(y - 1) = \] 

\[ x^2 + 2x = \]  
\[ 2x(y - 4) + 3y(4 - y) = \] 

2) difference of squares:
   \( (x - y)(x + y) = \) 

\[ (2x + y)(2x - y) = \]  
\[ (5x - 4y)(4y + 5x) = \] 

\[ (1 - xy)(1 + xy) = \] 

Factor Difference of Squares:
\[ x^2 - 36 = \]  
\[ x^3 - 4x = \] 

\[ 12x^2y^2 - 27y^3 = \]  
\[ x^4 - 16 = \]  
\[ x^2 + 9 = \]
3) perfect squares: 

\[(x + y)^2 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}\]

\[(x - y)^2 = \underline{\hspace{2cm}}\]

\[(3x+y)^2 = \underline{\hspace{2cm}}\]

\[(4x+y)^2 = \underline{\hspace{2cm}}\]

\[(2x-5y)^2 = \underline{\hspace{2cm}}\]

Factor Perfect Squares:

\[x^2 + 12x + 36 = \underline{\hspace{2cm}}\]

\[x^2 - 8x + 64 = \underline{\hspace{2cm}}\]

\[x^2 + 10x + 25 = \underline{\hspace{2cm}}\]

\[x^2 - 4x - 4 = \underline{\hspace{2cm}}\]

\[x^3 - 6x^2 + 9x = \underline{\hspace{2cm}}\]

Find the missing term so that the resulting polynomial is a perfect square.

\[x^2 + 12x + \underline{\hspace{2cm}}\]

\[x^2 - 8x + \underline{\hspace{2cm}}\]

\[x^2 + \underline{\hspace{2cm}} + 25\]

\[x^2 - \underline{\hspace{2cm}} + 49\]

\[4x^2 + 8x + \underline{\hspace{2cm}}\]

\[9x^2 - 12x + \underline{\hspace{2cm}}\]
4) sum – difference of cubes:
sopps: s: square of the first, o: opposite sign, p: product of the terms, p: plus, s: square of the last term

\[(x - y)(x^2 + xy + y^2) = \quad \]_________________________

\[(2x + y)(4x^2 - 2xy + y^2) = \quad \]_________________________

\[(2 - 3y)(4 + 6y + 9y^2) = \quad \]_________________________

Factor
Sum/Difference of Cubes:

\[x^3 - 8 = \quad \]_________________________
\[x^3 + 27 = \quad \]_________________________

\[8x^3 + 64 = \quad \]_________________________
\[2x - 16xy^3 = \quad \]_________________________

\[x^3 + y^6 = \quad \]_________________________

\[1 - 64x^6 = \quad \]_________________________
Some more examples.
5) **Trinomials of the form** $ax^2 + bx + c$

$(2x - 3)(x + 2) = \underline{\phantom{0000}}$

$(4x + y)^2 = (4x + y)(4x + y) = \underline{\phantom{0000000}}$

$(2x - 5)(3x + 4) = \underline{\phantom{00000000}}$

**Factoring Trinomials of the form** $ax^2 + bx + c$

**Case I:** $a = 1$, factor $x^2 + bx + c$

+ $c$: tells you whether the signs are the same or different in the factors ( ) ( )
  if $c > 0$, they are the same; if $c < 0$, they are different

+ $b$: tells you which of the products are positive and which are negative

+ $c$: also tells you what numbers to work with (factors of $c$)

$x^2 + 6x + 5 = \underline{\phantom{00000}} \quad \quad x^2 + 8x + 7 = \underline{\phantom{00000}}$

$x^2 - 8x + 15 = \underline{\phantom{00000}} \quad \quad x^2 - 6x + 8 = \underline{\phantom{00000}}$

$x^2 - 4x - 3 = \underline{\phantom{00000}} \quad \quad x^2 - 2x - 48 = \underline{\phantom{00000}}$

$x^2 + 3x - 54 = \underline{\phantom{00000}} \quad \quad x^2 - 12x - 28 = \underline{\phantom{00000}}$

$x^2 + 4x + 124 = \underline{\phantom{00000}} \quad \quad x^4 + 12x^2 + 32 = \underline{\phantom{0000000}}$
Case II: If $a \neq 0$, factor $ax^2 + bx + c$

Same ideas as before (Case I) but now we have to worry about the factors of $a$.

\[
2x^2 + 5x + 2 = \underline{} \\
3x^2 - 8x + 4 = \underline{}
\]

\[
4x^2 - 4x - 3 = \underline{} \\
4x^2 + 2x - 6 = \underline{}
\]

\[
12x^2 - x - 11 = \underline{} \\
4x^4 + 3x^2 - 1 = \underline{}
\]
Grouping:

2x + 3xy → answer: __________________

rewrite and then factor:

3(x - 2) + x(x - 2) = ____________

(x + y)(x + 3) - z(x + 3) = ______________

2xy - 3x - 4y + 6 = ______________________

x^2 - y^2 + 2y - 1 = ______________________

x^3 - y^3 + x - y = ______________________
Review of methods of factoring.
Factoring: process of writing a polynomial as a product of other polynomials of equal or lesser degree.

Methods:
- GCF – always look for a common factor - 1st method
- Difference of squares: \( x^2 - y^2 = (x - y)(x + y) \)
- Sum – Difference of Cubes: \( x^3 + y^3 = (x + y)(x^2 - xy + y^2) \), \( x^3 - y^3 = (x - y)(x^2 + xy + y^2) \) -- SOPPS
- Perfect Squares: \( x^2 + 2xy + y^2 \), first and last must always be positive

trinomials:

grouping:

Other Examples:
Multiply \((x + 2y)^2 = \) \( (3x - 4y)^2 = \)

More Factoring examples:
\( x^2 - 8x + 64 = \) \( x^2 + 10x + 25 = \)
\( x^2 + 4xy + 4y^2 = \) \( 4x^2 - 12xy + 9y^2 = \)
\( x^2 - 16x - 16 = \)

Grouping –
\( 2(x - y) - y(x - y) = \) \( xy + 2x - y^2 - 2y = \)
\( x^2 - y^2 - 2y - 4 = \) \( x^3 - y^3 - x + y = \)
Problem set #7
(Rational Expressions) Algebraic Fractions -- quotients of polynomials.

ex. \(2x / (x^2 - 3x + 4)\), \((x - 4) / 3\), \((x^2 + 1) / (x^2 + 2x + 1)\), \(\frac{3}{4}\), …

Similar rules as when we work with real fractions: \(\frac{1}{2} + \frac{1}{4} = \ldots\), \(\frac{1}{2} \cdot \frac{3}{5} = \ldots\)

Find

\[
\frac{3}{28} \div \frac{9}{16} = \ \ \ \frac{12}{25} + \frac{3}{25} = \ \ \ \frac{3}{12} - \frac{7}{8} =
\]

Algebraic Fraction (rational expression)

consists of a quotient of polynomials

\[
\frac{3}{8} - 4 \ \ \ \frac{3}{x} + 2 \ \ \ \frac{x + 2}{x - 1} \ \ \ \frac{2x - 1}{x^3 + 2x + 5}
\]

Domain of a rational expression:
values of \(x\) that are allowed in the expression

\[
\frac{x}{x + 3} \rightarrow \text{we can use any value of } x \text{ except } x = 3,
\]

So we say that the domain of this algebraic fraction is all real numbers except \(x = 3\)

What is the domain of each of the following

\[
\frac{x}{x + 2} \rightarrow \frac{3x - 1}{x^2 - 4} \rightarrow \frac{x + 2}{x^2 - 3x - 15}
\]
Simplifying fraction
Sometimes we can reduce fractions to equivalent fractions with a “smaller” denominator as well as being able to perform algebraic operations on these fractions: add, subtract, multiply, divide, ….

ex. Reduce each of the following fractions to lowest terms →

a) \[ \frac{x-1}{x^2-1} = \] b) \[ \frac{2x+1}{8x^2+4x} = \]

c) \[ \frac{x^2-2x+4}{x^4+8x} = \]

Multiplication / Division

a) \[ \frac{x^2+x}{x^2-1} \cdot \frac{2x-2}{4x+8} = \]

b) \[ \frac{x^3+1}{x^2-x+1} \div \frac{x^2+x}{2x} = \]

c) \[ \frac{x^2-2x-3}{x^2-x-2} \div \frac{x^2-4x+3}{3x^2-3} = \]
Addition/Subtraction

a) \( \frac{2x-4}{x^2+5} + \frac{4x-3}{x^2+5} = \) \\

b) \( \frac{4x+4}{2x+3} - \frac{x+5}{2x+3} = \) \\

c) \( \frac{x-7}{x+3} - \frac{3x-1}{x+3} = \) \\

d) \( \frac{x-1}{4x} + \frac{2}{x} = \) \\

e) \( \frac{2x+1}{2x^2} - \frac{3}{x} = \) \\

f) \( \frac{x+1}{x-1} - \frac{x-1}{x+2} = \)
g) \[ \frac{3 + x}{x + 1} - \frac{x + 2}{x^2 - 1} = \]

h) \[ \frac{2}{x - 2} + \frac{2 - 2x}{2 - x} = \]

i) \[ \frac{2/ - 1/2}{3/ \sqrt{x} + 1/2} = \]

j) \[ \frac{1}{x - 1} - \frac{1}{3 - \frac{1}{x - 1}} = \]

\[ 1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{x}}} = \]
Solving Equations

\[ x + 2x = x + 4: \text{ solution set } \_\_\_\_\_\_\_ \quad x - 2 = 0 \Rightarrow \text{ solution set } \_\_\_\_\_\_\_ \]

These two equations have the same solution. Two equations are said to be equivalent provided they have the same solution set.

Reduce equations to an equivalent form whose solution is easy to read – we may use the following two operations

1) You can add (subtract) any quantity to both sides of an equation to obtain an equivalent equation

\[ x + 4 = 3 \Rightarrow \_\_\_\_\_\_\_ \quad x - 5 = -3 \Rightarrow \_\_\_\_\_\_\_ \]

2) You can multiply (divide) both sides of an equation by any nonzero value

\[ 2x = \frac{1}{2} \Rightarrow \_\_\_\_\_\_\_ \quad x/4 = -\frac{2}{3} \Rightarrow \_\_\_\_\_\_\_ \]

Other Examples -

1) \[ 2x - 4 = 3 \Rightarrow \_\_\_\_\_\_\_ \]  \quad 2) \[ x/4 + 6 = 1 \Rightarrow \_\_\_\_\_\_\_ \]

3) \[ 3n/5 - n = -2 \Rightarrow \_\_\_\_\_\_\_ \]  \quad 4) \[ 2 - 3(1 - 2x) = 1 \Rightarrow \_\_\_\_\_\_\_ \]
More examples:

1) 8/89. Sally took four tests in science class. On each successive test, her score improved by 3 points. If her mean score was 69.5%, what did she score on the first test?

2) 12/89 A college student earns $20 per day delivering advertising brochures door-to-door, plus 75 cents for each person he interviews. How many people did he interview on a day when he earned $56.

3) 16/89 A builder wants to install a triangular window with the angles shown below. What angles will he have to cut to make the window fit?

\[ <\Lambda = 30^\circ, <B = 10^\circ + x, \text{ and } <C = 2x \]
Quadratic equations –

An equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation in $x$.
When written in that order with $a > 0$, we say it is in standard form.

Examples:
Write in standard form and find $a$, $b$, $c$

$x^2 = 5x - 2 \Rightarrow a = _____ \ b = _______ \ c = _______

$1 - x^2 = 2x \Rightarrow a = _____ \ b = _______ \ c = _______

To find the solution of equations of this form we may use one of the following methods:

1) factoring: either $ax^2 + bx = 0$, or $ax^2 + bx + c = 0$
2) square root method: $ax^2 + c = 0$
3) completing the square: any equation of the form $ax^2 + bx + c = 0$
4) Quadratic formula: any equation of the form $ax^2 + bx + c = 0$
Test I review:

- Know sets of numbers – natural #'s, whole #'s, integers, rational, irrational, real, prime, nonnegative integers, positive integers
  - be able to classify a number as a member of a particular set
  - be able to give an example of a number that is a member of one set but not another
  - compare sets, know how they differ in terms of the elements (numbers) in the set

- Properties of sets –
  - commutative law of addition, commutative law of multiplication, associative law of addition, associative law of multiplication, distributive law

- Properties and definitions of numbers
  - absolute value, GCF, LCM
  - inequalities on a number line
  - order of operations: PEMDAS
  - scientific notation
  - evaluating expressions

- Basic operations
  - be able to add, subtract, multiply, divide any real number

- Exponents
  - know the five basic properties
  - work with positive integer exponents, exponent of zero, negative coefficients or signs, negative exponents, fractional exponent

- Radicals –
  - be able to simplify and work problems as done in class and over homework, fractional exponents and radicals

- Polynomials –
  - basic operations - add, subtract, multiply, divide,
  - degree, terms, factors, classify – monomials, binomials, trinomials,
  - long-hand division

- Factor polynomials; GCF, difference of squares, perfect squares, sum of cubes, difference of cubes, trinomials, grouping
  - make sure to work with any combination of the above methods

- Algebraic fractions: add, subtract, multiply, divide,
  - complex fractions

- Linear equations in one variable - solve, find solution set
  - solve for a variable in a formula
  - word problems

- Identify equations as linear, quadratic equations
  - state methods of solutions for quadratic equations
  - Write a quadratic equation in its standard form, identify a, b, c
Write down quadratic formula

Rectangular Coordinate System (Cartesian Coordinate System)

Construction: x-axis, y-axis, quadrants, origin

Plotting points: x-coordinate (abscissa), y-coordinate (ordinate)
Plot the points A(2, -3), B(4, 0), C(-3, 0), D(0, -2)

Intercepts: x-intercept, y-intercept
## Table of Contents – Part I (Notes 1)

<table>
<thead>
<tr>
<th>Page Range</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 4</td>
<td>Test Review</td>
</tr>
<tr>
<td>5 - 7</td>
<td>Sets of real numbers</td>
</tr>
<tr>
<td>8</td>
<td>Practice problem set #1</td>
</tr>
<tr>
<td>9 - 10</td>
<td>Properties of real numbers</td>
</tr>
<tr>
<td>11</td>
<td>Practice problem set #2</td>
</tr>
<tr>
<td>12 - 14</td>
<td>GCF – different ways to find</td>
</tr>
<tr>
<td>15 - 16</td>
<td>LCM – different types of examples</td>
</tr>
<tr>
<td>17 - 18</td>
<td>Absolute value</td>
</tr>
<tr>
<td>19</td>
<td>Problem Set #3</td>
</tr>
<tr>
<td>20</td>
<td>Inequalities: basic notation</td>
</tr>
<tr>
<td>21 - 22</td>
<td>Terminology – for polynomials</td>
</tr>
<tr>
<td>23</td>
<td>Practice Problem set #4 (includes-simple exponents)</td>
</tr>
<tr>
<td>24 - 29</td>
<td>Review of exponents and their rules</td>
</tr>
<tr>
<td>30</td>
<td>Scientific Notation</td>
</tr>
<tr>
<td>31</td>
<td>Problem set #5</td>
</tr>
<tr>
<td>32 - 33</td>
<td>Radicals</td>
</tr>
<tr>
<td>34</td>
<td>Fractional Exponents</td>
</tr>
<tr>
<td>35 - 39</td>
<td>More on radicals</td>
</tr>
<tr>
<td>40</td>
<td>Problem Set #6</td>
</tr>
<tr>
<td>41 - 42</td>
<td>Polynomials</td>
</tr>
<tr>
<td>43 - 50</td>
<td>Products and Factoring (polynomials)</td>
</tr>
<tr>
<td>51</td>
<td>Problem set #7</td>
</tr>
<tr>
<td>52 - 55</td>
<td>Algebraic (Rational) Fractions</td>
</tr>
<tr>
<td>56</td>
<td>Problem set #8</td>
</tr>
<tr>
<td>57 - 58</td>
<td>Solving linear equations</td>
</tr>
<tr>
<td>59</td>
<td>Quadratic equations - standard form</td>
</tr>
<tr>
<td>60</td>
<td>Problem set #9</td>
</tr>
<tr>
<td>61</td>
<td>Test review – outline of topics</td>
</tr>
<tr>
<td>56</td>
<td>Rectangular coordinates, plotting points, graphing lines</td>
</tr>
</tbody>
</table>